Supermodularity, Comparative Advantage, and Global Supply Chains*

Yan Ma†

March 3, 2017

Abstract

We develop a North-South model with many tasks to investigate how task asymmetry and task complementarity affect the South getting involved in global supply chains. Task complementarity exists if the production function exhibits supermodularity. We demonstrate that the South is excluded from global supply chains if the production process exhibits supermodularity and tasks are symmetric. When tasks are asymmetric, there is a trade-off between the supermodularity effect and the comparative advantage effect to determine whether some tasks will be offshored to the South. If the comparative advantage effect dominates the supermodularity effect, there exist gains from trade in tasks, which is complementary to the "productivity effect" in Grossman and Rossi-Hansberg (2008). A chain of comparative advantage can be defined based on the skill sensitivity of the production process. Moreover, we demonstrate that the chain of comparative advantage may vary with the number of tasks offshored, due to complementarity between tasks. This contrasts with the predictions of standard Ricardian model, which only depends on comparative advantage.

*This paper was started when I visited Vanderbilt University. I am indebted to Eric W. Bond for his valuable advice and support. Valuable comments from Arnaud Costinot, Fumio Dei, Taiji Furusawa, Gene Grossman, Jota Ishikawa, Peter Neary, Joel Rodrigue, and conference participants at Asian Pacific Trade seminars 2014, Society of Advanced Economic Theory 2015, Fall 2015 Midwest International Trade Meetings, and Workshop at Kagawa University 2016 are greatly appreciated. Financial support from JSPS KAKENHI Grant (No. 16H03617) is greatly acknowledged.

†Graduate School of Business Administration, Kobe University, 2-1, Rokkodai, Nada, Kobe 657-8501, Japan. E-mail: mayan003@kobe-u.ac.jp.

1 Introduction

As noted by Grossman and Rossi-Hansberg (2008), the nature of trade has changed dramatically from an exchange of complete goods to an exchange of tasks. There is abundant evidence that trade entails different countries adding value to global supply chains.[1] Casual observation suggests that global supply chains differ with regard to the involvement of developing countries, i.e. the South. For instance, the South is involved in the production of the Barbie doll as in Feenstra (1998).[2] However, the South is hardly involved in the production of main parts of 787 dreamliner as in Grossman and Rossi-Hansberg (2012).[3] Why do global supply chains differ with regard to the involvement of developing countries? To answer this question, we examine how characteristics of tasks (task asymmetry and task complementarity) affect offshoring in a setup with many tradeable tasks.

Our study is inspired by Grossman and Rossi-Hansberg (2008), in which a new paradigm of trade, i.e., trade in tasks is developed. As noted in their paper (p 1995), "But we have not incorporated the possibility that some subset of tasks carried out by a given factor are especially complementary to a particular subset of those disposed by another. Such circumstance can arise when the technology requires certain groups of tasks to be performed in close proximity." An example provided by Grossman and Rossi-Hansberg (2008, p 1995) is that the tasks performed by a nurse during surgery are most valuable when the surgeon is nearby. As noted by Milgrom and Roberts (1990), the notion of complementarity corresponds to a property of the production function known as supermodularity.[4] In this paper, we introduce task complementarity into a setup of offshoring by using a general supermodular production function.

In order to investigate how task asymmetry affects offshoring, we consider two cases. First, we consider the case of symmetric tasks, i.e., each task plays an equal important role in the production process. Next, we consider the case that tasks are asymmetric, i.e., some task contributes more to the output than other tasks. We demonstrate that the South is

[1] See Hummels et. al (2001), Baldwin (2006) for empirical evidence.

[2] "Of the $2 export value for the dolls when they leave Hong Kong for the United States, about 35 cents covers Chinese labor, 65 cents covers the cost of materials [which are imported from Taiwan, Japan, and the United States], and the remainder covers transportation and overhead, including profits earned in Hong Kong" (Feenstra (1998), p. 36).

[3] "The wings are produced in Japan, the engines in the United Kingdom and the United States, the flaps and ailerons in Canada and Australia, the fuselage in Japan, Italy and the United States, the horizontal stabilizers in Italy, the landing gear in France, and the doors in Sweden and France." (Grossman and Rossi-Hansberg (2012), p. 1)

[4] Costinot (2009) introduces the supermodularity into the international context.

excluded from global supply chains if the production process exhibits supermodularity and tasks are symmetric. When tasks are asymmetric, there is a trade-off between the supermodularity and comparative advantage to determine whether the South can get involved in the global supply chain. If the comparative advantage effect dominates the supermodularity effect, the South can participate in the global supply chain. Due to the nature of asymmetric tasks, absolute productivity differences are a source of comparative advantage between countries in our paper. In addition, if the South get involved in the global supply chain, we demonstrate that there exists the gains from trade in tasks, which is different from the "productivity effect" in Grossman and Rossi-Hansberg (2008). Moreover, we demonstrate that the chain of comparative advantage may vary with the number of tasks offshored, due to complementarity between tasks. This contrasts with the predictions of standard Ricardian model, which only depends on comparative advantage.

Our paper is related to several strands of literature. First, our paper is related to the literature of fragmentation, which is large and diverse.[5] As noted by Baldwin and Venables (2013, page 246), the production process with fragmentation consists of two types. One we refer to as the "spider": multiple limbs (parts) coming together to form a body (assembly), which may be the final product itself or a component (such as a module in the auto-industry). The other is the "snake": the good moving in a sequential manner from upstream to downstream with value added at each stage. Seminal papers of Antras and Chor (2013), Costinot, Vogel, and Wang (2013), and Dei (2010) consider a "snake" type of production process. Studies of Feenstra and Hanson (1996), Levine (2010), Ngienthi, Ma, and Dei (2011), and Ngienthi and Ma (2011) investigate a "spider" type of production process. In this paper, we develop a setup including not only both types of production processes but also the mixed type of production process to investigate how task asymmetry and task complementarity affect offshoring.

Our "snake" type of production shares a similar environment to Sobel (1992), Kremer (1993), and Costinot, Vogel, and Wang (2013) in the sense that production is sequential and subject to mistakes. Both our paper and Costinot, Vogel, and Wang (2013) do so in an open-economy setup. However, our paper focuses on how task asymmetry and task complementarity affect the global supply chain, while Costinot, Vogel, and Wang (2013) focus on labor assignment along the global supply chain.

Next, our paper is related to a growing literature using matching and assignment models

[5] See Antras and Rossi-Hansberg (2009) for a review.

in an international context. For example, Grossman and Maggi (2000), Grossman (2004), Yeaple (2005), Ohnsorge and Trefler (2007), Nocke and Yeaple (2008), Costinot (2009), and Costinot, Vogel and Wang (2013). In this paper, the assignment of tasks to workers (countries) exhibits positive assortative matching, i.e., more productive workers (countries) are assigned to relatively skill sensitive tasks. When tasks are symmetric, positive matching excludes the South from the global supply chain. When tasks are asymmetric, if the comparative advantage effect is sufficiently strong, the South may get involved in the global supply chain.

Our paper is organized as follows. In section 2, we introduce the supermodular production function as in Milgrom and Roberts (1990). We obtain unit costs under different modes of production in Section 3. In section 4, we first investigate the possibility of offshoring with a linear production function, which can be interpreted as a discrete task setup used in Grossman and Rossi-Hansberg (2008). Next, we examine the possibility of offshoring when tasks are symmetric and asymmetric. The role of task asymmetry is examined in section 5. In section 6, we investigate how task complementarity affects the offshoring pattern. Section 7 concludes. All proofs are in Appendix.

2 Production Processes, Tasks, and Supermodular Function

In this section, we introduce definitions used in this paper.

2.1 Tasks

We assume that the production process of each firm involves N tasks. The tasks are indivisible and each task must be performed by exactly one worker. Let $f(q_1, q_2, \cdots, q_N)$ represent the output when task k ($k = 1, 2, \cdots, N$) is performed by a worker with skill level q_k and vector $\mathbf{q} \equiv (q_1, q_2, \cdots, q_N) \in R^N$ represent the skill vector. We denote by $\mathbf{q}_{-k}$ the vector $\mathbf{q}$ with the kth component removed and by $\mathbf{q}_{-kj}$ the vector $\mathbf{q}$ with the kth and jth components removed.

Definition 1: If all tasks are symmetric, we have $f(\mathbf{q}_{-kj}, q_k, q_j) = f(\mathbf{q}_{-kj}, q'_k, q'_j)$ for any two tasks k, j, where $k, j = 1, \cdots, N$, $k \neq j$, $q_k = q'_j$, and $q_j = q'_k$.

One example is $f(q_1, q_2) = (q_1^\theta + q_2^\theta)^{\frac{1}{\theta}}$, $\theta < 1$ and $\theta \neq 0$, used by Grossman and Maggi (2000).

Definition 2: If there exist tasks k and j, $k \neq j$, and $k, j = 1, \cdots, N$, such that $f(\mathbf{q}_{-kj}, q_k, q_j) \neq f(\mathbf{q}_{-kj}, q'_k, q'_j)$ for $q_k \neq q_j$, $q_k = q'_j$ and $q_j = q'_k$, we say that tasks are asymmetric.

One example is $f(q_1, q_2) = (\alpha q_1^\theta + q_2^\theta)^{\frac{1}{\theta}}$, $\alpha > 1$, $\theta < 1$, and $\theta \neq 0$. Clearly, if $q_1 \neq q_2$, $f(q_1, q_2) \neq f(q_2, q_1)$. Intuitively, changing the workers' assigned tasks leads to a change in output.

Definition 3: If $f(\mathbf{q}_{-kj}, q_k, q_j) > f(\mathbf{q}_{-kj}, q'_k, q'_j)$ for $q_k = q'_j > q_j = q'_k$, we say that task k is relatively more skill sensitive than task j. If tasks are asymmetric and we have $f(\mathbf{q}_{-kj}, q_k, q_j) = f(\mathbf{q}_{-kj}, q'_k, q'_j)$ for $k \neq j$, $q_k = q'_j$ and $q_j = q'_k$, we say that task k and task j are equally skill sensitive.

For example, in the case of $f(q_1, q_2) = (\alpha q_1^\theta + q_2^\theta)^{\frac{1}{\theta}}$, $\alpha > 1$, $\theta < 1$, and $\theta \neq 0$, since $f(q_1, q_2) > f(q_2, q_1)$ with $q_1 > q_2$, we say task 1 is skill-sensitive task. Intuitively, task 1 and task 2 can be explained as the tasks performed by a doctor and a nurse, respectively. In the case of $f(q_1, q_2, q_3) = (q_3^\theta + \alpha q_1^\theta q_2^\theta)^{\frac{1}{\theta}}$, $\alpha > 1$, $\theta < 1$, and $\theta \neq 0$, tasks are asymmetric while task 1 and task 2 are equally skill sensitive.

2.2 Production Process

In this paper, we consider three types of production process: the "snake" type, the "spider" type, and the mixed type of "snake" and "spider". As in Baldwin and Venables (2013), the "spider" type of production process refers to the production process that multiple tasks come together to form a product and a mistake occurs at one task does not affect the performance of other tasks. For example, the production function $f(q_1, q_2) = (q_1^\theta + q_2^\theta)^{\frac{1}{\theta}}$, $\theta < 1$, $\theta \neq 0$, is an example of "spider" type of production process with two symmetric tasks. The production function $f(q_1, q_2) = (\alpha q_1^\theta + q_2^\theta)^{\frac{1}{\theta}}$, $\theta < 1$, $\theta \neq 0$, represents an example of "spider" type of production process with two asymmetric tasks.

The "snake" type of production process (eg. Costinot, Vogel, and Wang (2013)) refers to the sequential production process which is subject to mistakes.[6] When a mistake occurs at one task, the performance of all tasks before that task is entirely lost. For this type

[6] In order to focus on characteristics of tasks, we assume that each task is conducted by exactly one worker. However, in order to focus on the labor assignment along the supply chain, the production function with a discrete number of stages in Costinot, Vogel, and Wang (2013), is shown by $q(s+1) = e^{-\lambda_c} \min[q(s), l(s+1)]$, where $q(s)$ and $l(s+1)$ are the inputs used in stage $s+1$ (see footnote 3 in their paper). λ_c is an exogenous technological charateristic of country c, which represents the probability that a mistake occurs along the supply chain.

of production process, skill level q_k reflects the probability of performing the kth task successfully or $1 - q_k$ reflects the probability that a mistakes occurs at kth task. For example, the "O-ring" production function $f(q_1, q_2) = q_1 q_2$ in Kremer (1993) is an example of "snake" type of production process with two symmetric tasks. The production function $f(q_1, q_2) = q_2^2 q_1$, used in Kremer and Maskin (2006) and Dei (2010), represents the "snake" type of production process with two asymmetric tasks.

If the production process includes both "snake" type and "spider" type of production processes, we call it the mixed type of production process. For example, the production function $f(q_1, q_2, q_3) = [q_1^\theta + (q_2 q_3)^\theta]^{\frac{1}{\theta}}$ is an example of the mixed type of production process.

2.3 Supermodular Function

We assume the production process is supermodular. Following Milgrom and Roberts (1990, p.516), we can express this as:

Assumption 1: A function $f : R^n \to R$ is supermodular if for all $\mathbf{q}$, $\mathbf{q\prime} \in R^n$,

$$f(\mathbf{q}) + f(\mathbf{q}') \leq f(\min(\mathbf{q}, \mathbf{q\prime})) + f(\max(\mathbf{q}, \mathbf{q\prime})).$$

Let $\mathbf{q}$, $\mathbf{q\prime} \in R^n$. We say that $\mathbf{q} \geq \mathbf{q}'$ if $q_k \geq q'_k$ for all k. Define $\max\{\mathbf{q}, \mathbf{q}'\}$ to be the point in R^n whose kth component is $\max\{q_k, q'_k\}$, and $\min\{\mathbf{q}, \mathbf{q}'\}$ to be the point whose kth component is $\min\{q_k, q'_k\}$.

Milgrom and Roberts (1990) pointed out that whether a function exhibiting supermodularity can be easily checked by Theorem 2, which states that a smooth function f is supermodular if and only if $f_{kj} \geq 0$ for $k \neq j$. $f_{kj} \geq 0$ implies that the marginal product of the skill level in one task is increasing in the skill level used to perform another task. Put differently, supermodularity is equivalent to complementarity between two tasks. Examples of the "spider" and "snake" production functions above exhibit supermodularity.

3 Production Function and Unit Costs

Let's consider homogeneous firms in a perfectly competitive market. We assume that the output increases in the skill level of each worker, i.e., $f_k(q_1, q_2, \cdots, q_N) > 0$, $k = 1, 2, \cdots, N$, and constant returns to scale in the number of workers. In order to focus on how task asymmetry and task complementarity affect the possibility of offshoring, we assume that labor in each country is homogeneous. Moreover, we assume that the skill level of labor in the North, q, is higher than that of labor in the South, q^*, that is,

$$q > q^* > 0.$$

The Northern labor is served as the numeraire so that the Northern wage rate is set at 1. Denote by w^* the Southern wage rate. We focus on one industry and hence take the Southern wage w^* as given.

In the North, firms have an opportunity to offshore tasks to the South. The following cost functions can be applied to any type of production process. The unit cost function when a northern firm does not offshore any task, that is, the unit cost function of a Northern local firm is

$$c(\mathbf{q}) = \frac{N}{f(\mathbf{q})}. \tag{1}$$

$f(\mathbf{q}) = f(q, \cdots, q)$, where $\mathbf{q} \equiv (q, q, \cdots, q)$, represents the output when all tasks are performed in the North. The unit cost function when all tasks are performed in the South is

$$c(\mathbf{q}^*, w^*) = \frac{Nw^*}{f(\mathbf{q}^*)}. \tag{2}$$

Similarly, $f(\mathbf{q}^*) = f(q^*, \cdots, q^*)$, where $\mathbf{q}^* \equiv (q^*, \cdots, q^*)$, represents the output when all tasks are performed in the South. The unit cost function when a Northern firm offshores i tasks, where $i = 1, 2, \cdots, N-1$, to the South is

$$c(q_{(N-i)}, q^*_{(i)}, w^*) = \frac{(N-i) + iw^*}{f(q_{(N-i)}, q^*_{(i)})}, \tag{3}$$

where $f(q_{(N-i)}, q^*_{(i)})$ represents the output when a Northern firm offshores i tasks to the South.[7]

4 The Possibility of Offshoring

In this section, we consider the possibility of offshoring. We first investigate the case of a linear production function, which can be interpreted as a discrete-task setup used in Grossman and Rossi-Hansberg (2008). Next, we examine the case when all tasks are

[7] In this paper, offshoring means that the production is performed in two locations.

symmetric. Finally, we consider the case when tasks are asymmetric.

4.1 Linear Production Function

We consider the possibility of offshoring under a linear production function:

$$f(q_1, \cdots, q_N) = \sum_{k=1}^{N} f_k \cdot q_k,$$

where f_k, $k = 1, 2, \cdots, N$, is fixed. We assume that tasks are ranked in strictly decreasing skill sensitivity, i.e., $f_1 > f_2 > \cdots > f_N$. The 1st task is the most skill-sensitive task and the Nth task is the least skill-sensitive task. Thus, the cutoff task of offshoring, j, is determined by

$$p_N \cdot f_j \cdot (q - q^*) = w - w^*,$$

where p_N denotes the price of this product when all tasks are performed in the North. The left-hand side and the right-hand side of the above equation represent the benefit and the cost of using labors with a higher skill level, respectively. It is clear that the South has a comparative advantage in conducting the least skill-sensitive tasks (from the jth to the Nth tasks).

Lemma 1: Under linear production function, northern firms will offshore the least skill-sensitive tasks to the South.

Under linear production function, the offshoring pattern totally depends on comparative advantage. If we reinterpret $q - q^*$ as the offshoring cost, we obtain a discrete-task setup of Grossman and Rossi-Hansberg (2008). In the setup of Grossman and Rossi-Hansberg (2008), the production function is a Leontief production function, i.e., a continuum of tasks performed by a single factor of production must be conducted exactly "once" to produce a unit of output of a good. A Northern firm producing good j that performs task k abroad requires $a_{Lj}\beta t_j(k)$ units of southern labor, where β is a shift parameter to represent the technology for offshoring, and $t_j(k)$ refers to the cost of offshoring task k in sector j with $t'_j(k) > 0$. Tasks are ordered in strictly increasing offshoring costs and since both country use the same amounts of a factor to perform a task, a reduction in β leads to more tasks offshored to the South. In our setup with discrete tasks, a_{Lj} is equal to 1. f_k plays similar roles as $t_j(k)$ to rank tasks though f_k decreases in k. $q - q^*$ plays a similar role as β but it affects output instead of the cost. Thus, a sufficient increase in q^*, i.e., a reduction in the offshoring cost $q - q^*$, with w^* fixed leads to a fall in j, i.e., more tasks are offshored to the South. It follows that we have a similar "productivity effect" as in Grossman and Rossi-Hansberg (2008).[8]

4.2 The Possibility of Offshoring with Symmetric Tasks

We now turn to the case of supermodular production function with symmetric tasks. We illustrate the unit cost lines in Figure 1. Since tasks are symmetric, the unit costs of offshoring any i ($i = 1, 2 \cdots, N-1$) tasks are the same.

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Figure 1: Unit Costs

Given i ($i = 1, 2, \cdots, N-1$), a Northern firm will decide to offshore i tasks to the South if the unit cost under offshoring is the lowest. Let $w_1^*(i)$ represent a Southern wage, at which the unit cost if i tasks are offshored is equal to that if all tasks are performed in the South. Let $w_2^*(i)$ represent a Southern wage, at which the unit cost of a Northern local firm is equal to that if i tasks are offshored to the South.

In Figure 1, if $w_2^*(i) > w_1^*(i)$, offshoring i tasks can occur in the interval of $[w_1^*(i), w_2^*(i)]$, because offshoring offers the minimum unit cost. If $w_1^*(i) > w_2^*(i)$, offshoring will not occur as dashed line AB in Figure 1 shows.

From (2) and (3), we have

$$w_1^*(i) = \frac{(N-i) \cdot f(\mathbf{q}^*)}{N \cdot f(q_{(N-i)}, q_{(i)}^*) - i \cdot f(\mathbf{q}^*)}. \tag{4}$$

Similarly, from (1) and (3), we have

$$w_2^*(i) = \frac{N \cdot f(q_{(N-i)}, q_{(i)}^*) - (N-i) \cdot f(\mathbf{q})}{i \cdot f(\mathbf{q})}. \tag{5}$$

From (4) and (5), we obtain

$$w_2^*(i) - w_1^*(i) = \frac{Nf(q_{(N-i)}, q_{(i)}^*)[Nf(q_{(N-i)}, q_{(i)}^*) - (N-i)f(\mathbf{q}) - i \cdot f(\mathbf{q}^*)]}{i \cdot f(\mathbf{q})[Nf(q_{(N-i)}, q_{(i)}^*) - i \cdot f(\mathbf{q}^*)]}. \tag{6}$$

[8] Note that $p_N = \frac{Nw}{q\sum_{i=1}^N f_i}$. When I tasks are offshored, i.e., $j = N - I + 1$, the price of the product becomes $p_O = \frac{(N-I)w + Iw^*}{q\sum_{i=1}^{N-I} f_i + q^* \sum_{i=N-I+1}^N f_i}$. Since $I \cdot f_{N-I+1} \geq \sum_{i=N-I+1}^N f_i$, we have $p_N > p_O$. It follows that p_O decreases in I.

Let define $\gamma(i,q,q^*) \equiv Nf(q_{(N-i)}, q^*_{(i)}) - (N-i)f(\mathbf{q}) - i \cdot f(\mathbf{q}^*)$. The sign of (6) depends on the sign of $\gamma(i,q,q^*)$, because $Nf(q_{(N-i)}, q^*_{(i)}) - i \cdot f(\mathbf{q}^*) > 0$. If $\gamma(i,q,q^*) > 0$, then $w_2^*(i) > w_1^*(i)$, which means that offshoring can occur. If $\gamma(i,q,q^*) \leq 0$, then $w_2^*(i) \leq w_1^*(i)$ and thus offshoring cannot occur. We demonstrate in Appendix A that if the production process exhibits supermodularity, we have $\gamma(i,q,q^*) \leq 0$ for any $i < N$. Therefore, if the production function is supermodular and tasks are symmetric, offshoring from the North to the South will never occur.

Proposition 1 *If the production process exhibits supermodularity and tasks are symmetric, a Northern firm will never offshore any tasks to the South, that is, the South will never be involved in this global supply chain.*

This proposition is consistent with the idea that supermodularity promotes self-matching. When tasks are symmetric, self-matching excludes the South from some global supply chains.

4.3 The Possibility of Offshoring with Asymmetric Tasks

In this section, we discuss the possibility of offshoring with asymmetric tasks. As in Section 4.1, we assume that tasks are ranked in strictly decreasing skill sensitivity, i.e., the 1st task is most skill-sensitive task and the Nth task is the least skill-sensitive task.

If a Northern firm can only offshore one task, which task will it offshore? The relative unit cost of offshoring task k to offshoring task $j(\neq k)$, $\frac{c(\mathbf{q}_{-k},q^*,w^*)}{c(\mathbf{q}_{-j},q^*,w^*)}$, is obtained as follows:

$$\frac{c(\mathbf{q}_{-k},q^*,w^*)}{c(\mathbf{q}_{-j},q^*,w^*)} = \frac{f(\mathbf{q}_{-j},q^*)}{f(\mathbf{q}_{-k},q^*)}.$$

$f(\mathbf{q}_{-k},q^*)$ represents the output if the kth $(k = 1,2,\cdots,N-1)$ task is offshored to the South. If tasks are symmetric, $f(\mathbf{q}_{-j},q^*)$ is equal to $f(\mathbf{q}_{-k},q^*)$. It follows that $\frac{c(\mathbf{q}_{-k},q^*,w^*)}{c(\mathbf{q}_{-j},q^*,w^*)}$ is equal to 1. Therefore, offshoring any task is indifferent. However, when tasks are asymmetric, following our definition of skill sensitivity in Section 2, we have $f(\mathbf{q}_{-1},q^*) < f(\mathbf{q}_{-2},q^*) < \cdots < f(\mathbf{q}_{-N},q^*)$. Thus, if the kth task is relatively skill sensitive than the jth task, we have $f(\mathbf{q}_{-j},q^*) > f(\mathbf{q}_{-k},q^*)$, which leads to $c(\mathbf{q}_{-k},q^*,w^*) > c(\mathbf{q}_{-j},q^*,w^*)$. Therefore, offshoring the jth task is more cost-saving than offshoring the kth task. It follows that

$$c(\mathbf{q}_{-1}, q^*, w^*) > \cdots > c(\mathbf{q}_{-k}, q^*, w^*) > \cdots > c(\mathbf{q}_{-N}, q^*, w^*).$$

Clearly, if a Northern firm offshores only one task to the South, it will offshore the least skill-sensitive task.

Next, we turn to the case of offshoring any given i $(1 < i < N)$ tasks. Let set $I(i)$, $1 \leq i < N$, represent the least skill-sensitive combination of i tasks, i.e., offshoring that combination of i tasks leads to a minimum cost among offshoring all possible combinations of i tasks. Clearly, $I(1)$ consists of the least sensitive task, the Nth task. Since the ranking of tasks' skill sensitivity may depend on the location of other tasks or task complementarity, $I(i) \subset I(j)$ may not always hold for any $i < j$, which will be discussed later. For now, we rerank tasks such that the least skill-sensitive combination of i tasks are ranked from the $(N - i + 1)$th task to the Nth task. Hence, $f(q_{(N-i)}, q^*_{(i)})$ represents the output if the least skill-sensitive combination of i tasks, i.e., $I(i)$ are offshored. We use the notion of $f(q_{(N-i-m)}, q^*_{(i)}, q_{(m)})$, where $1 \leq m \leq N - i$, to represent the output if any other i tasks except $I(i)$ are offshored. It follow that $f(q_{(N-i)}, q^*_{(i)}) > f(q_{(N-i-m)}, q^*_{(i)}, q_{(m)})$ for any $N - i \geq m \geq 1$ and

$$\frac{c(q_{(N-i)}, q^*_{(i)}, w^*)}{c(q_{(N-i-m)}, q^*_{(i)}, q_{(m)}, w^*)} = \frac{f(q_{(N-i-m)}, q^*_{(i)}, q_{(m)})}{f(q_{(N-i)}, q^*_{(i)})} < 1.$$

Thus, the South has a comparative advantage in conducting $I(i)$.

Proposition 2 *If tasks are asymmetric and Northern firms can offshore i tasks to the South, it will offshore the least skill-sensitive combination of i tasks, $I(i)$, to the South.*

Because of the nature of asymmetric tasks, the source of comparative advantage in our paper is absolute productivity difference in the form of uniform skill level at all tasks, regardless of types of production process. This shares the same logic with Costinot, Vogel, and Wang (2013).[9]

[9] In Costinot, Vogel and Wang (2013), the sequential nature of production leads to the asymmetry of production stages, i.e., the later the production stage is, the more skill sensitive it is.

4.4 Supermodularity and Comparative Advantage

What determines whether Northern firms can offshore i tasks to the South? Note that we still obtain (4), (5), and (6) when tasks are asymmetric. The sign of (6) also depends on the sign of $\gamma(i, q, q^*) \equiv N f(q_{(N-i)}, q^*_{(i)}) - (N-i) f(\mathbf{q}) - i \cdot f(\mathbf{q}^*)$, where $f(q_{(N-i)}, q^*_{(i)})$ represents the output if the least skill-sensitive combination of i tasks, i.e., $I(i)$ are offshored. If $\gamma(i, q, q^*) > 0$, the unit cost line of a offshoring firm in Figure 1 is below the point of C, i.e., Northern firms can offshore i tasks to the South.[10]

We demonstrate in Appendix B that if the production process exhibits supermodularity, for any given N and i, we have

$$\gamma(i, q, q^*) \equiv N f(q_{(N-i)}, q^*_{(i)}) - [(N-i) f(\mathbf{q}) + i \cdot f(\mathbf{q}^*)] \leq \varphi(i, t),$$

where t satisfies $t = N - ki$, $i \geq t > 0$, if $[\frac{N}{i}] = \frac{N}{i}$, we have $k \equiv [\frac{N}{i}] - 1$; otherwise, $k \equiv [\frac{N}{i}]$, and

(1) if $i = ct$, $c \geq 1$ is an integer,

$$\begin{aligned} \varphi(i, t) \equiv (N-t) f(q_{(N-i)}, q^*_{(i)}) &- t \sum_{m=0}^{c-1} \sum_{l=2}^{k} f(q_{(N+mt-li)}, q^*_{(i)}, q_{((l-1)i-mt)}) \\ &- t \sum_{m=1}^{c} f(q^*_{(mt)}, q_{(N-i)}, q^*_{(i-mt)}); \end{aligned}$$

[10] Note that this result holds whether tasks are ranked in strictly decreasing skill sensitivity or not.

(2) if $c_1 t < i < (c_1+1)t$, $c_j = [\frac{ji}{t}]$, $j = 0, 1, 2, \cdots, s$, $s = \min\{\ x \mid [\frac{xi}{t}] = \frac{xi}{t}\}$,

$$\begin{aligned}
\varphi(i,t) \equiv (N - \frac{t}{x}) f(q_{(N-i)}, q^*_{(i)}) - \frac{t}{x} \sum_{m=0}^{c_1} \sum_{l=2}^{k} f(q_{(N+mt-li)}, q^*_{(i)}, q_{((l-1)i-mt)}) \\
- \frac{t}{x} \sum_{j=1}^{x-2} \sum_{m=c_j+1}^{c_{j+1}} \sum_{l=2+j}^{k} f(q_{(N+mt-li)}, q^*_{(i)}, q_{((l-1)i-mt)}) \\
- \frac{t}{x} \sum_{j=1}^{x} \sum_{m=c_{j-1}+1}^{c_j} f(q^*_{(mt-(j-1)i)}, q_{(N-i)}, q^*_{(ji-mt)}) \\
- \frac{t}{x} \sum_{m=c_{s-1}+1}^{c_s-1} \sum_{l=x+1}^{k} f(q_{(N+mt-li)}, q^*_{(i)}, q_{((l-1)i-mt)}) \\
- \frac{t}{x} \sum_{j=1}^{x-1} \sum_{m=c_j+1}^{c_s} f(q_{(mt-ji)}, q^*_{(i)}, q_{(N-mt+(j-1)i)}).
\end{aligned}$$

In both (1) and (2), each negative term represents the output when a combination of i tasks are offshored to the South. For example, $f(q^*_{(mt)}, q_{(N-i)}, q^*_{(i-mt)})$ in (1) reflects the output when tasks from 1st to mtth and from $(i-mt)$th to Nth are offshored to the South (the total number of offshored tasks is $mt + (i-mt) = i$). In both (1) and (2), since the total number of negative terms is equal to the number of terms that consist of $f(q_{(N-i)}, q^*_{(i)})$, $\varphi(i,t)$ reflects the South's comparative advantage.[11] When tasks are symmetric, $\varphi(i,t)$ is equal to zero. It follows that $\gamma(i, q, q^*) \leq 0$. Hence, offshoring will never occur. When tasks are asymmetric, $\varphi(i,t)$ is greater than zero. Thus, offshoring may occur.

It is clear that there are two forces at work in determining whether offshoring is possible. On the one hand, the supermodularity of the production process, i.e., the complementarity between tasks promotes self-sorting, which is called the supermodularity effect. On the other hand, task asymmetry leads to the South having a comparative advantage in the least skill-sensitive combination of tasks. We call this force as the comparative advantage effect. When tasks are symmetric, the comparative advantage effect disappears and hence the supermodularity effect prevents the South from getting involved in the global supply chain. When tasks are asymmetric, if the comparative advantage effect dominates the supermodularity effect, the South may get involved into some global supply chains. In

[11] For example, in (1), on the one hand, the number of terms consisting of $f(q_{(N-i)}, q^*_{(i)})$ is $N - t$, which is equal to ki. On the other hand, the total number of negative terms is $tc(k-1) + ct = ctk$, which is equal to ki because $i = ct$.

particular, under the linear production function as in Section 4.1, the supermodularity effect disappears and offshoring only depends on the comparative advantage effect.[12]

Proposition 3 *If tasks are asymmetric and the production process exhibits supermodularity, there is a tradeoff between the supermodularity effect and the comparative advantage effect to determine the possibility of offshoring. If the comparative advantage effect dominates the supermodularity effect, offshoring the least skill-sensitive combination of tasks to the South becomes possible.*

Let us examine an example with a supermodular production function and two asymmetric tasks. In this case, $\gamma(1, q, q^*) \equiv 2f(q, q^*) - f(q, q) - f(q^*, q^*)$. If the first task is relatively skill-sensitive than the second one, according to Appendix B, we have

$$2f(q, q^*) \leq f(q, q) + f(q^*, q^*) + \varphi(1, 1),$$

where $\varphi(1, 1) = f(q, q^*) - f(q^*, q) > 0$ represents the comparative advantage effect. Hence, if the comparative advantage effect dominates the supermodularity effect, offshoring may occur. Note if tasks are symmetric, we have $f(q, q^*) = f(q^*, q)$ and $\varphi(1, 1) = 0$, i.e., the comparative advantage effect disappears.

Proposition 3 holds for all types of production process. Under the "snake" type of production process such as $f(q_1, q_2) = q_1^2 q_2$, a skill-insensitive task (task 2) can be offshored to the South. Under the "spider" type of production process such as $f(q_1, q_2) = (\alpha q_1^\theta + q_2^\theta)^{\frac{1}{\theta}}$, $\alpha > 1$, $\theta < 1$, and $\theta \neq 0$, the region in which offshoring the skill-insensitive task (task 2) becomes possible is shown by "Offshoring Region" in Figure 2.[13] Clearly, the "Offshoring Region" shows the region in which the comparative advantage effect dominates the supermodularity effect.

[12] Under linear production function as in Section 3, we have $\gamma(N - j, q, q^*) \equiv Nf(q_{(j)}, q^*_{(N-j)}) - j \cdot f(\mathbf{q}) - (N - j) \cdot f(\mathbf{q}^*) > 0$, for the cutoff task j.

[13] For simplicity, q is nornalized as 1. $\gamma(\alpha, \theta, q^*) = 0$ in Figure 2 is equivalent to $\gamma(1, 1, q^*) \equiv 2[\frac{\alpha + (q^*)^\theta}{\alpha + 1}]^{\frac{1}{\theta}} - 1 - q^* = 0$.

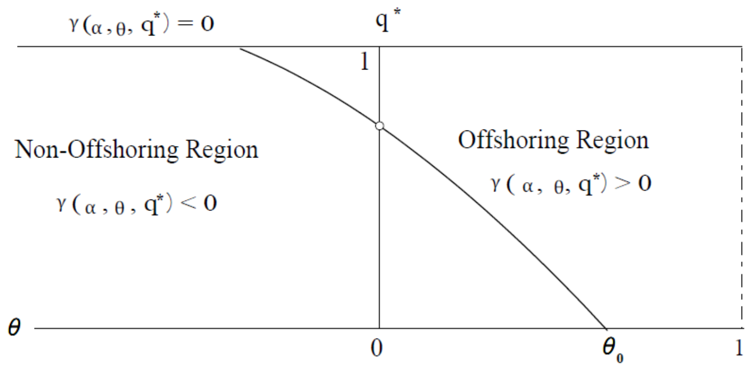

Figure 2: Asymmetric Tasks and Offshoring

4.5 Tasks Offshored at Equilibrium

We have discussed that there are two forces determining whether offshoring a least skill-sensitive combination of i tasks, $I(i)$, is possible. Proposition 3 implies that when the comparative advantage effect dominates the supermodularity effect, there exists a non-empty interval $[w_1^*(i), w_2^*(i)]$ and offshoring $I(i)$ is possible in that interval. In the following, we investigate how many tasks are offshored in the equilibrium.

Given the skill level in the North q and that in the South q^*, and the Southern wage w^*, the equilibrium production pattern of tasks leads to the minimum unit cost among unit costs of offshoring $I(i)$, $i = 1, 2, \cdots, N-1$, i.e., $c(q_{(N-i)}, q^*_{(i)}, w^*)$. Let $\hat{\imath}$ and $c(q_{(N-\hat{\imath})}, q^*_{(\hat{\imath})}, w^*)$ represent the number of tasks offshored and the unit cost of offshoring in the equilibrium, respectively. We have

Proposition 4 *Given q, q^*, and w^*, the equilibrium production pattern of tasks will satisfy that $\gamma(\hat{\imath}, q, q^*) > 0$ and $c(q_{(N-\hat{\imath})}, q^*_{(\hat{\imath})}, w^*) = \min\{c(q_{(N-i)}, q^*_{(i)}, w^*),\ i = 1, 2, \cdots N-1\}$.*

It is worth noting that the equilibrium production pattern of tasks does not always include the largest number of tasks among those combinations satisfying $\gamma(i, q, q^*) > 0$, $i = 1, 2, \cdots N-1$. Here combination refers to least skill-sensitive combination of tasks. There is a possibility that there exist two combinations: one includes i tasks and the other includes $j(\neq i)$ tasks, and $\gamma(k, q, q^*) > 0$, $k = i, j$, such that their corresponding intervals $[w_1^*(k), w_2^*(k)]$ are non-empty. For example, $f(q_1, q_2, q_3) = (\alpha_1 q_1^\theta + \alpha_2 q_2^\theta + q_3^\theta)^{\frac{1}{\theta}}$, $\alpha_1 > \alpha_2 > 1$,

$\theta < 1$, $\theta \neq 0$. If one task is offshored, task 3 is offshored and its unit cost is

$$c(q, q, q^*, w^*) = \frac{2 + w^*}{[\alpha_1 q^\theta + \alpha_2 q^\theta + (q^*)^\theta]^{\frac{1}{\theta}}}.$$

If two tasks are offshored, task 2 and task 3 are offshored and the unit cost is

$$c(q, q^*, q^*, w^*) = \frac{1 + 2w^*}{[\alpha_1 q^\theta + \alpha_2 (q^*)^\theta + (q^*)^\theta]^{\frac{1}{\theta}}}.$$

For simplicity, let $q = 1$, $q^* = 0.81$, $\alpha_1 = 3$, $\alpha_2 = 2$, and $\theta = \frac{1}{2}$. The intervals of $[w_1^*(1), w_2^*(1)]$ and $[w_1^*(2), w_2^*(2)]$ are shown in Figure 3.[14] Clearly, if the Southern wage is higher than w_d^*, at which offshoring one task (task 3) and offshoring two tasks (task 2 and 3) are indifferent, offshoring task 3 is optimal. If the Southern wage is lower than w_d^*, offshoring task 2 and 3 is optimal.

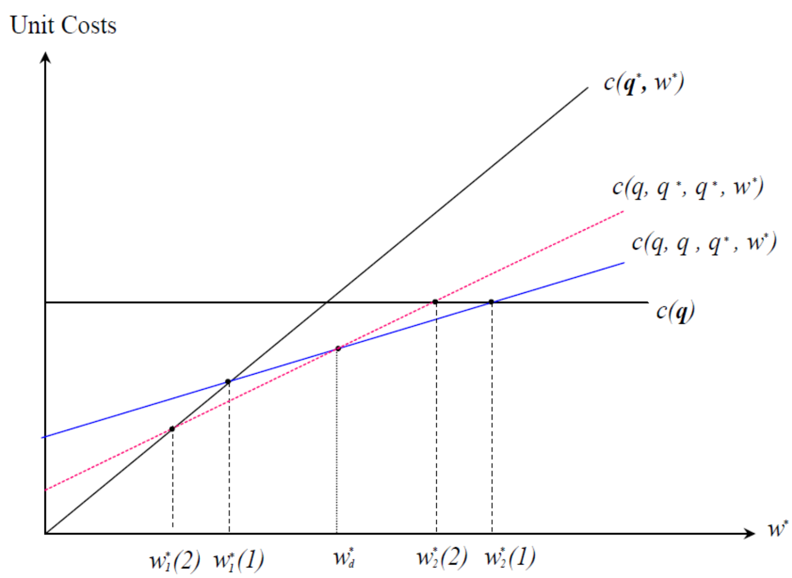

Figure 3: Tasks Offshored in the Equilibirum

Let $\hat{\imath}$ represent the number of tasks offshored in the equilibrium. If offshoring occurs, we have $\gamma(\hat{\imath}, q, q^*) \equiv Nf(q_{(N-\hat{\imath})}, q^*_{(\hat{\imath})}) - (N - \hat{\imath})f(\mathbf{q}) - \hat{\imath} \cdot f(\mathbf{q}^*) > 0$. We call $Nf(q_{N-\hat{\imath}}, q^*_{\hat{\imath}}) - (N-\hat{\imath})f(\mathbf{q}) - \hat{\imath} \cdot f(\mathbf{q}^*)$ the gains from trade in task. $Nf(q_{N-\hat{\imath}}, q^*_{\hat{\imath}}) - (N-\hat{\imath})f(\mathbf{q}) - \hat{\imath} \cdot f(\mathbf{q}^*) > 0$

[14] Using (4) and (5), if task 3 is offshored, we have $w_1^*(1) = 0.774811$, $w_2^*(1) = 0.900833$. Similarly, if task 2 and 3 are offshored, we obtain $w_1^*(2) = 0.7444828$, and $w_2^*(2) = 0.85375$.

implies that offshoring the combination of $\hat{\imath}$ least skill sensitive tasks $I(\hat{\imath})$ leads to a larger output than producing $(N-\hat{\imath})$ units in the North and $\hat{\imath}$ units in the South. Put it differently, offshoring $I(\hat{\imath})$ to the South leads to a cost saving, which is much the same as would result from an economy-wide increase in the productivity of labor. Note that this gains from trade in task is different from the "productivity effect" in Grossman and Rossi-Hansberg (2008), which is owing to a fall in offshoring costs.[15] However, the gains from trade in task in our setup come from the occurrence of offshoring. In addition, with w^* fixed, a rise in q^* such that more tasks can be offshored to the South also brings about a "productivity effect" in our setup.

5 The Role of Task Asymmetry

In this section, we examine how task asymmetry affects the possibility of offshoring. We define an increase in task asymmetry to occur if the output from offshoring a subset of the tasks increases relative to that obtained from locating all of the tasks in the same country. This can be expressed formally as

Definition 4: The degree of task asymmetry increases if $\frac{f(q_{(N-i)},q^*_{(i)})}{f(\mathbf{q})}$ and/or $\frac{f(q_{(N-i)},q^*_{(i)})}{f(\mathbf{q}^*)}$ increase.

$f(q_{(N-i)}, q^*_{(i)})$ represents the output if the least skill-sensitive combination of i tasks, $I(i)$ are offshored. A higher task asymmetry means that the productivity of performing tasks in $I(i)$ is improved.

It was shown previously that offshoring i tasks will be profitable if $w^* \in [w_1^*(i), w_2^*(i)]$. We say that a change in the production process makes offshoring more likely if it expands the interval of foreign wages for which offshoring is profitable. Using (4) and (5), we obtain

$$w_2^*(i) - w_1^*(i) = \frac{Nf(q_{(N-i)}, q^*_{(i)})/f(\mathbf{q}) - (N-i)}{i} - \frac{N-i}{Nf(q_{(N-i)}, q^*_{(i)})/f(\mathbf{q}^*) - i}. \quad (7)$$

A rise in $f(q_{(N-i)}, q^*_{(i)})/f(\mathbf{q}^*)$ leads to a fall in $w_1^*(i)$ and a rise in $f(q_{(N-i)}, q^*_{(i)})/f(\mathbf{q})$ brings about an increase in $w_2^*(i)$. The following result is then immediate:

Proposition 5 *A higher task asymmetry makes offshoring more likely.*

[15] As noted by Grossman and Rossi-Hansberg (2008, footnote 14), only when offshoring occured already, a reduction in offshoring cost leads to "productivity effect".

Let's consider an example $f(q_1, q_2) = (\alpha q_1^\theta + q_2^\theta)^{\frac{1}{\theta}}$, $\alpha > 1$, $\theta < 1$, and $\theta \neq 0$. A higher α implies a higher task asymmetry. The output if the skill-insensitive task (task 2) offshored to the South is given by $f(q_1, q_2^*) = (\alpha q^\theta + q^{*\theta})^{\frac{1}{\theta}}$. It is clear that both $\frac{f(q_1,q_2^*)}{f(q_1,q_2)}$ and $\frac{f(q_1,q_2^*)}{f(q_1^*,q_2^*)}$ increase in α, which follows that $w_2^*(1) - w_1^*(1)$ increases in α.[16] This is because a rise in task asymmetry, α, leads to a stronger comparative advantage effect.[17] Thus, a rise in task asymmetry leads to a larger offshoring region in Figure 4.

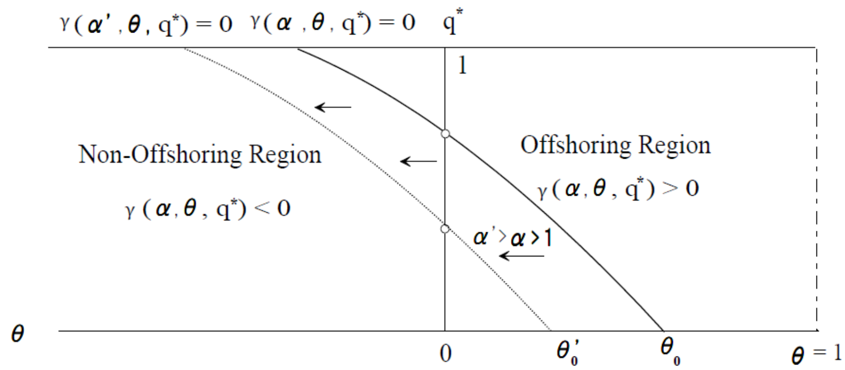

Figure 4: Role of Task Asymmetry

5.1 Technical Progress and Task Asymmetry

Technical progress leads to a higher productivity in performing some or all tasks. We consider two types of technical progresses: one is task technical progress, the other is skill technical progress. If the productivity of conducting each task is increased by the same proportion with one skill (q or q^*) , we say that a task-neutral technical progress occurs. We say that a skill-neutral technical progress occurs if the productivity of performing a task with all skills (both q and q^*) is increased by the same proportion. Thus, a skill-task-neutral technical progress means that the productivity of conducting each task with all skills is increased by the same proportion.

We examine how technical progresses affect task asymmetry and offshoring. Let's first explain two types of technical progresses with an example of $f(q_1, q_2) = (\alpha_1 q_1^\theta + \alpha_2 q_2^\theta)^{\frac{1}{\theta}}$, $\theta < 1$, $\theta \neq 0$, $\alpha_j = \alpha_j(s)$, $j = 1, 2$, and $s \in \{q, q^*\}$. If $d\alpha_j(q) = d\alpha_j(q^*) = d\alpha_k(q) = d\alpha_k(q^*) > 0$, for $j, k = 1, 2$, $j \neq k$, we say that a skill-task-neutral technical progress occurs.

[16] Clearly, $\gamma(\alpha, \theta, q^*) = 2(\alpha + q^{*\theta})^{\frac{1}{\theta}} - (\alpha + 1)^{\frac{1}{\theta}} - (\alpha q^{*\theta} + q^{*\theta})^{\frac{1}{\theta}}$ increases in α, i.e., $\frac{\partial \gamma(\alpha,\theta,q^*)}{\partial \alpha} > 0$.

[17] The comparative advantage effect is $\varphi(1, 1) = f(q, q^*) - f(q^*, q) = (\alpha q^\theta + q^{*\theta})^{\frac{1}{\theta}} - (\alpha q^{*\theta} + q^\theta)^{\frac{1}{\theta}}$, which clearly increases in α.

If $d\alpha_1(s) = d\alpha_2(s) > 0$ for $s = q$ or q^*, we say that a task-neutral but a skill-biased technical progress occurs. For example, if $d\alpha_1(q) = d\alpha_2(q) > d\alpha_1(q^*) = d\alpha_2(q^*)$, we say that a task-neutral but a high-skill-biased technical progress occurs. If $d\alpha_j(q) = d\alpha_j(q^*) > 0$ for $j = 1$ or 2, we say that a skill-neutral but a task-biased technical progress occurs. For example, if $d\alpha_j(q) = d\alpha_j(q^*) > d\alpha_k(q) = d\alpha_k(q^*)$, $j, k = 1, 2$, $j \neq k$, we say that a skill-neutral but a task j biased technical progress occurs. Clearly, the above explanation can be extended to production function with N tasks such as $f(q_1, q_2, \cdots, q_N) = (\sum_{j=1}^{N} \alpha_j q_j^{\theta})^{\frac{1}{\theta}}$, where $\alpha_j = \alpha_j(s)$, $j = 1, 2 \ldots, N$, and $s \in \{q, q^*\}$.

How does a technical progress affect offshoring? Suppose that tasks are symmetric before the technical progress occurs, i.e., $\alpha_1(s) = \alpha_2(s)$. It is clear that a skill-task-neutral technical progress will not bring about an occurrence of offshoring because tasks are symmetric too after such technical progresses. If a skill-neutral but a j task-biased technical progress occurs, for example, $d\alpha_j(q) = d\alpha_j(q^*) > d\alpha_k(q) = d\alpha_k(q^*) = 0$, offshoring task k becomes possible because task k becomes relatively less sensitive to skill as shown by Figure 2. Suppose that $\alpha_j(s) > \alpha_k(s)$, $j, k = 1, 2$, $j \neq k$, before the technical progress. The skill-neutral but task j biased technical progress such as $d\alpha_j(q) = d\alpha_j(q^*) > d\alpha_k(q) = d\alpha_k(q^*) = 0$ enlarges the region of offshoring task k because both $\frac{f(q_j, q_k^*)}{f(q_j, q_k)}$ and $\frac{f(q_j, q_k^*)}{f(q_j^*, q_k^*)}$ increase. However, if a skill-neutral but task k biased technical progress, i.e., $0 = d\alpha_j(q) = d\alpha_j(q^*) < d\alpha_k(q) = d\alpha_k(q^*)$ occurs, the region of offshoring task k shrinks because both $\frac{f(q_j, q_k^*)}{f(q_j, q_k)}$ and $\frac{f(q_j, q_k^*)}{f(q_j^*, q_k^*)}$ decrease. In addition, there is a possibility that task k is moved back to the North, i.e., "reshoring" becomes possible.

Proposition 6 *A skill-neutral but a sensitive-task biased technical progress may lead to offshoring or enlarge the offshoring region but a skill-neutral but an insensitive-task biased technical progress may lead to "reshoring".*

A skill-neutral but a sensitive-task biased technical progress enlarges the offshoring region because it increases the degree of task asymmetry. However, a skill-neutral but an insensitive-task biased technical progress decreases the degree of task asymmetry and thus it may lead to "reshoring".

We turn to investigating how a skill-neutral but a task-biased technical progress affects the offshoring. Suppose that task 2 is relatively skill insensitive before the technical progress, i.e., $\alpha_1(s) > \alpha_2(s)$. Let's consider a task-neutral but high-skill biased technical progress such as $d\alpha_1(q) = d\alpha_2(q) > d\alpha_1(q^*) = d\alpha_2(q^*) = 0$. Such a technical progress leads to an increase in $\frac{f(q_1,q_2^*)}{f(q_1^*,q_2^*)}$ but a fall in $\frac{f(q_1,q_2^*)}{f(q_1,q_2)}$. As a result, the effect of such a technical progress on the region of offshoring task 2 is ambiguous. Similarly, a task-neutral but a low-skill biased technical progress such as $0 = d\alpha_1(q) = d\alpha_2(q) < d\alpha_1(q^*) = d\alpha_2(q^*)$ has an ambiguous effect on the offshoring region, because $\frac{f(q_1,q_2^*)}{f(q_1^*,q_2^*)}$ decreases while $\frac{f(q_1,q_2^*)}{f(q_1,q_2)}$ rises.

5.2 Standardization

In most industries, production processes become more standardized as goods become mature over time. According to Vernon (1966, page 7), when a new product is initially introduced, the product itself may be quite unstandardized for a time; its inputs, its processing, and its final specifications may cover a wide range.[18] Thus, when a new product is initially introduced, all tasks are equally sensitive to skill. When the production process become more standardized, some tasks become less sensitive to skill, for example, assembly. Within our framework, we refer to "standardization" as more tasks becoming less sensitive to skill over time. To examine the potential implications of this particular type of technological progress, we consider the productivity of performing those tasks with low skill (q^*) is increased more than that with high skill (q), which may be due to learning by doing in the South as noted by Bond and Ma (2013).

Let's go back to the above example of $f(q_1, q_2) = (\alpha_1 q_1^\theta + \alpha_2 q_2^\theta)^{\frac{1}{\theta}}$, $\theta < 1$, $\theta \neq 0$, and $\alpha_j = \alpha_j(s)$, $j = 1, 2$, and $s \in \{q, q^*\}$. When the product is initially introduced, $\alpha_1(s) = \alpha_2(s)$ and tasks are symmetric. As analyzed in Section 4.2, Northern firms will never offshore to the South. When the product becomes mature due to product developments such as $d\alpha_1(q) = d\alpha_1(q^*) = d\alpha_2(q^*) > d\alpha_2(q)$, offshoring task 2 may become possible because $\frac{f(q_1,q_2^*)}{f(q_1,q_2)}$ increases. These results are reminiscent of Vernon (1966) "product cycle hypothesis".[19]

6 Task Complementarity and Offshoring Pattern

We will investigate how task complementarity affects the offshoring pattern. To do so, we focus on how the chain of comparative advantage vary with the number of tasks offshored.

[18] Contrast the great variety of automobiles produced and marketed before 1910 with the thoroughly standardized product of the 1930's, or the variegated radio designs of the 1920's with the uniform models of the 1930's. (See Vernon (1966), p 7)

[19] See also Grossman and Helpman (1991), Antras (2005), and Bond and Ma (2013).

We start with the offshoring pattern when one more task is offshored. We firstly assume that $f_i(\cdot)$ strictly decreases in i, i.e., $f_1(\cdot) > \cdots > f_N(\cdot)$, which guarantees that $I(i)$ consists of the least sensitive i tasks.[20] In other words, we exclude the possibility that the ranking of task sensitivity depend on the location of other tasks or task complementarity. It follows that $I(1) \subset I(2) \subset \cdots \subset I(N-1)$. It is clear that if offshoring one more task is possible, the next least skill-sensitive task will be offshored. One example is the linear production function discussed in Section 4.1.

Interestingly, if we relax the assumption that $f_i(\cdot)$ strictly decreases in i, i.e., tasks are ranked in strictly decreasing skill sensitivity, we can see that there is a possibility that $I(1) \subset I(2) \subset \cdots \subset I(N-1)$ will not hold. Put it differently, if we allow the ranking of task sensitivity depend on the location of other tasks or task complementarity, the chain of comparative advantage may vary with the number of tasks offshored, due to complementarity between tasks. For example, the production function $f(q_1, q_2, q_3) = q_3 + \eta q_1 q_2$, where $\eta > 1$, does not satisfy the above assumption. We first show that if the Northern firm offshores one task, it will offshore the third task. For simplicity, we assume that $q \equiv 1$ and $w \equiv 1$ as assumed in the above. If the third task is offshored, the unit cost is

$$c(q, q, q^*, w^*) = \frac{2 + w^*}{q^* + \eta}.$$

Since offshoring the first task and offshoring the second task are indifferent, the unit cost of offshoring either task, for example, the second task is

$$c(q, q^*, q, w^*) = \frac{2 + w^*}{1 + \eta q^*}.$$

Clearly, we have $c(q, q, q^*, w^*) < c(q, q^*, q, w^*)$. Hence, the third task is the least skill sensitive task. Following the discussion in Section 3, whether task 3 is offshored or not depends on the sign of $\gamma(1, 1, q^*)$. We have

$$\begin{aligned}\gamma(1, 1, q^*) &= 3f(q, q, q^*) - 2f(\mathbf{q}) - f(\mathbf{q}^*) \\ &= (1 - q^*)(\eta + \eta q^* - 2).\end{aligned}$$

Clearly, if $\gamma(1, 1, q^*)$ is positive, which requires that $q^* > \frac{2}{\eta} - 1$, offshoring the third task

[20] See Appendix C for proof.

occurs.

Next, we turn to the case of offshoring two tasks. If the second and the third task are offshored, the unit cost is

$$c(q, q^*, q^*, w^*) = \frac{1 + 2w^*}{q^* + \eta q^*}.$$

If the first and the second tasks are offshored, the unit cost is

$$c(q^*, q^*, q, w^*) = \frac{1 + 2w^*}{1 + \eta (q^*)^2}.$$

Under the condition of $\eta q^* < 1$, we have $c(q, q^*, q^*, w^*) > c(q^*, q^*, q, w^*)$. That is, the least skill sensitive combination of two tasks is the first and the second tasks, i.e., $I(1) \subsetneq I(2)$. It follows

$$\begin{aligned}\gamma(2, 1, q^*) &= 3f(q^*, q^*, q) - f(\mathbf{q}) - 2f(\mathbf{q}^*)\\ &= (1 - q^*)(2 - \eta - \eta q^*).\end{aligned}$$

Hence, under the condition that $\eta q^* < 1$ and $q^* < \frac{2}{\eta} - 1$, we have $\gamma(2, 1, q^*) > 0$ and the first and second tasks are offshored together. Figure 4 shows the region of offshoring one task and that of offshoring two tasks under the condition of $\eta q^* < 1$. Lemma 2 below gives the necessary conditions for the chain of comparative advantage varying with the number of tasks offshored when the production process involves three tasks.

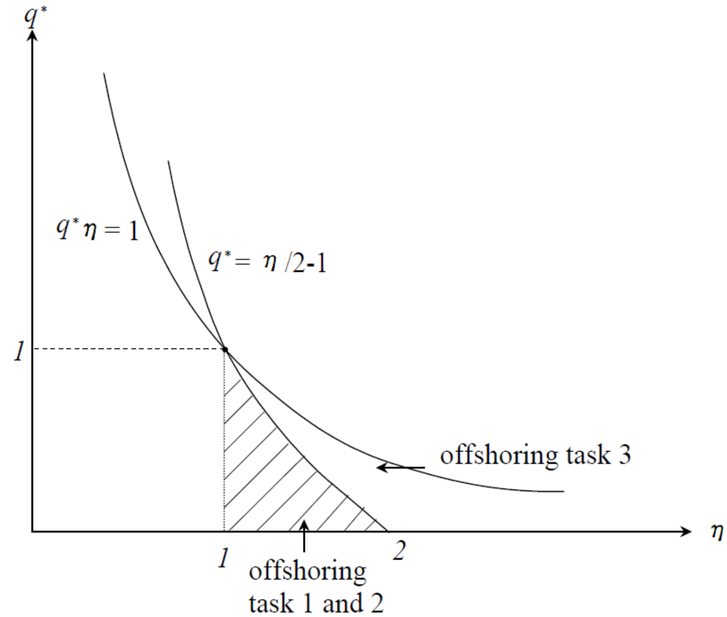

Figure 5: Offshoring Regions

Lemma 2: Suppose that the production of one unit of a good involves three tasks. When one task is offshored, the third task is offshored. When two tasks are offshored, the necessary conditions for offshoring task 1 and 2 are $f_{12} > f_{13}$ and $f_{12} > f_{23}$.

Proof: See Appendix D.

This Lemma considers the case that the production involves three tasks. Generally, we have the following Proposition.

Proposition 7 *When one task is offshored, task h is offshored. However, when two tasks are offshored, the necessary condition for task $i(\neq h)$ and $j(\neq h)$ being offshored at the same time are $f_{ij} > f_{ih}$ and $f_{ij} > f_{jh}$.*

Proof: See Appendix E.

Proposition 7 shows that when two tasks are offshored, the least sensitive task may not be offshored. Proposition 7 stands in sharp contrast to the predictions of standard Ricardian model, which only depends on the comparative advantage. Note if two tasks are offshored, the degree of complementarity between the two tasks offshored is not the maximum. For example, the production function $f(q_1, q_2, q_3) = q_1 + \delta q_2 q_3 + \eta q_4 q_5$, where $\eta > \delta > 1$. The degree of complementarity between task 4 and task 5 is highest. However, if the Northern firm offshores two tasks, under the condition of $\delta q^* > 1$, the second and third task will be offshored because they compose the least skill-sensitive combination of 2 tasks $I(2)$, which satisfies $\gamma(2, q, q^*) > 0$. Hence, when two tasks are offshored, task complementarity as well as comparative advantage determine the offshoring pattern.

7 Concluding Remarks

The nature of trade has changed from exchange of complete goods to the exchange of tasks, with different countries getting involved in global supply chains. Although trade in tasks and global supply chains have been discussed extensively, there is no study to examine how characteristics of tasks (task asymmetry and task complementarity) affect offshoring pattern. In this paper, we have developed a North-South setup to examine the effects of task asymmetry and task complementarity on offshoring pattern.

In this paper, we demonstrate that there is a tradeoff between the supermodularity effect and the comparative advantage effect to determine whether developing countries get involved in the global supply chain, regardless of types of production process. When tasks are symmetric, the comparative advantage effect disappear and hence the supermodularity effect, which promotes the positive assortative matching, exclude developing countries from global supply chains. When tasks are asymmetric, if the comparative advantage effect is sufficiently strong, developing countries may get involved in global supply chains. If developing countries get involved in the global supply chains, there exist gains from trade in tasks, which is complementary to the "productivity effect" in Grossman and Rossi-Hansberg (2008). Moreover, we demonstrate that the chain of comparative advantage may vary with the number of tasks offshored, due to complementarity between tasks. This contrasts with the predictions of standard Ricardian model, which only depends on comparative advantage.

Our setup developed here is useful to interpret several observations in the real world. However, different offshoring costs of tasks have abstracted from the current setup. This issue is left for our future work.

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Appendix A

We will prove that if the production process exhibits supmodularity and all tasks are symmetric, for any given N and $i < N$, we have

$$\gamma(i, q, q^*) = N \cdot f(q_{(N-i)}, q^*_{(i)}) - (N-i) \cdot f(\mathbf{q}) - i \cdot f(\mathbf{q}^*) \leq 0. \tag{AF}$$

Let $m \equiv N - i$. Thus, the above inequality becomes

$$\gamma(N-m, q, q^*) = N \cdot f(q_{(m)}, q^*_{(N-m)}) - m \cdot f(\mathbf{q}) - (N-m) f(\mathbf{q}^*) \leq 0. \tag{AM}$$

First, we will prove that for any given N, when $m = 1$, i.e., $i = N - 1$, (AM) holds. By using the definition of supmodularity, we have

$$f(q, q^*_{(N-1)}) + f(q^*_{(N-1)}, q) \leq f(\mathbf{q}^*) + f(q, q^*_{(N-2)}, q). \tag{A1}$$

The left-hand side of (A1) is equal to $2f(q, q^*_{(N-1)})$ because tasks are symmetric. Next, we use the definition of supermodularity for $f(q, q^*_{(N-2)}, q)$, which appears at the right-hand side of (A1). Since $f(q, q^*_{(N-2)}, q)$ is equal to $f(q_{(2)}, q^*_{(N-2)})$, we have

$$f(q_{(2)}, q^*_{(N-2)}) + f(q^*_{(N-1)}, q) \leq f(\mathbf{q}^*) + f(q_{(2)}, q^*_{(N-3)}, q), \tag{A2}$$

where $f(q_{(2)}, q^*_{(N-3)}, q) = f(q_{(3)}, q^*_{(N-3)})$. If we add (A1) and (A2), we have $3f(q, q^*_{(N-1)}) \leq 2f(\mathbf{q}) + f(q_{(3)}, q^*_{(N-3)})$. Note that $f(q, q^*_{(N-2)}, q)$ is cancelled out with $f(q_{(2)}, q^*_{(N-2)})$. We use the definition of supermodular production function recursively to obtain

$$f(q_{(3)}, q^*_{(N-3)}) + f(q^*_{(N-1)}, q) \leq f(\mathbf{q}^*) + f(q_{(4)}, q^*_{(N-4)}), \tag{A3}$$

$$\vdots$$

$$f(q_{(N-1)}, q^*) + f(q^*_{(N-1)}, q) \leq f(\mathbf{q}) + f(\mathbf{q}^*). \tag{AN}$$

Adding from (A1) to (AN) to yield

$$N \cdot f(q, q^*_{(N-1)}) \leq f(\mathbf{q}) + (N-1) \cdot f(\mathbf{q}^*).$$

The left-hand side is obtained because tasks are symmetric, i.e., $f(q, q^*_{(N-1)}) = f(q^*_{(N-1)}, q)$.

Thus, when $m = 1$, (AM) holds.

Suppose that when $m = N - 2$, (AM) holds. Thus, we have

$$N \cdot f(q_{(N-2)}, q^*_{(2)}) - (N-2) \cdot f(\mathbf{q}) - 2 \cdot f(\mathbf{q}^*) \leq 0. \tag{AG}$$

We will prove that when $m = N - 1$, i.e., $i = 1$, (AM) holds. By using the definition of supermodular production function, we have

$$f(q_{(N-1)}, q^*) + f(q^*, q_{(N-1)}) \leq f(\mathbf{q}) + f(q_{(N-2)}, q^*_{(2)}).$$

By using (AG), the above inequality becomes

$$f(q_{(N-1)}, q^*) \leq \frac{N-1}{N} f(\mathbf{q}) + \frac{1}{N} f(\mathbf{q}^*).$$

Thus, (AM) holds for $m = N - 1$. Therefore, (AF) holds for any given N and $i < N$.

Appendix B

We demonstrate that if the production function exhibits supermodularity, for any given N and i, we have

$$Nf(q_{(N-i)}, q^*_{(i)}) - [(N-i)f(\mathbf{q}) + i \cdot f(\mathbf{q}^*)] \leq \varphi(i,t),$$

where t satisfies $t = N - ki$, $i \geq t > 0$, if $[\frac{N}{i}] = \frac{N}{i}$, we have $k \equiv [\frac{N}{i}] - 1$; otherwise, $k \equiv [\frac{N}{i}]$, and

(1) if $i = ct$, $c \geq 1$ is an integer,

$$\varphi(i,t) \equiv (N-t)f(q_{(N-i)}, q^*_{(i)}) - t\sum_{m=0}^{c-1}\sum_{l=2}^{k} f(q_{(N+mt-li)}, q^*_{(i)}, q_{((l-1)i-mt)})$$
$$- t\sum_{m=1}^{c} f(q^*_{(mt)}, q_{(N-i)}, q^*_{(i-mt)});$$

When $m = c$, we have $f(q^*_{(mt)}, q_{(N-i)}, q^*_{(i-mt)}) = f(q^*_{(i)}, q_{(N-i)})$.

(2) $c_1 t < i < (c_1 + 1)t$, $c_1 \geq 1$, $c_j = [\frac{ji}{t}]$, $j = 0, 1, 2, \cdots, s$, $s > 1$, and $s = min\{\ x \mid$

$[\frac{xi}{t}] = \frac{xi}{t}\}$

$$\varphi(i,t) \equiv (N - \frac{t}{s}) f(q_{(N-i)}, q^*_{(i)}) - \frac{t}{s} \sum_{m=0}^{c_1} \sum_{l=2}^{k} f(q_{(N+mt-li)}, q^*_{(i)}, q_{((l-1)i-mt)})$$
$$- \frac{t}{s} \sum_{j=1}^{s-2} \sum_{m=c_j+1}^{c_{j+1}} \sum_{l=2+j}^{k} f(q_{(N+mt-li)}, q^*_{(i)}, q_{((l-1)i-mt)})$$
$$- \frac{t}{s} \sum_{j=1}^{s} \sum_{m=c_{j-1}+1}^{c_j} f(q^*_{(mt-(j-1)i)}, q_{(N-i)}, q^*_{(ji-mt)})$$
$$- \frac{t}{s} \sum_{m=c_{s-1}+1}^{c_s-1} \sum_{l=x+1}^{k} f(q_{(N+mt-li)}, q^*_{(i)}, q_{((l-1)i-mt)})$$
$$- \frac{t}{s} \sum_{j=1}^{s-1} \sum_{m=c_j+1}^{c_s} f(q_{(mt-ji)}, q^*_{(i)}, q_{(n-mt+(j-1)i)}).$$

Proof:

Since tasks are asymmetric, it is difficult to prove as in Appendix A. However, we follow the similar logic as in the symmetric task case to use the definition of supermodular production function recursively. We begin with adding $f(q_{(N-i)}, q^*_{(i)})$, in which the last i tasks are performed in the North, to $f(q_{(N-2i)}, q^*_{(i)}, q_{(i)})$, in which the last i tasks are performed in the South. Put it in another way, in order to obtain a $f(\mathbf{q})$ at the right-hand side, we match $q^*_{(i)}$ in $f(q_{(N-i)}, q^*_{(i)})$ with $q_{(i)}$ in $f(q_{(N-2i)}, q^*_{(i)}, q_{(i)})$, at the same time $q^*_{(i)}$ is put before $q_{(i)}$ in $f(q_{(N-2i)}, q^*_{(i)}, q_{(i)})$. The similar manipulating process is continued to obtain either a $f(\mathbf{q})$ or a $f(\mathbf{q}^*)$ at the right-hand side of each inequality. The manipulating process ends when we obtain both $f(\mathbf{q})$ and $f(\mathbf{q}^*)$ at the right-hand side. Since $q^*_{(i)}$ is shifted forward each time of the manipulating, we have two cases: case 1 is the case that N and i have a common divisor, and case 2 is the case that N and i do not have a common divisor.

(1) By using the definition of supermodular production function recursively, we have

$$f(q_{(N-i)}, q^*_{(i)}) + f(q_{(N-2i)}, q^*_{(i)}, q_{(i)}) \leq f(\mathbf{q}) + f(q_{(N-2i)}, q^*_{(2i)}) \tag{B1}$$

$$\vdots$$

$$f(q_{(N-(k-1)i)}, q^*_{((k-1)i)}) + f(q_{(N-ki)}, q^*_{(i)}, q_{((k-1)i)}) \leq f(\mathbf{q}) + f(q_{(N-ki)}, q^*_{(ki)}), \tag{BK-1}$$

Note that from (B1) to (BK-1), totally there are (k-1) inequalities with (Bj) responding to the jth inequality. Since $N - ki = t \leq i$, we have $f(q_{(N-ki)}, q^*_{(ki)}) = f(q_{(t)}, q^*_{(N-t)})$. It follows

$$f(q_{(t)}, q^*_{(N-t)}) + f(q^*_{(t)}, q_{(N-i)}, q^*_{(i-t)}) \leq f(\mathbf{q}^*) + f(q_{(N+t-i)}, q^*_{(i-t)}). \tag{Bt0}$$

If $c = 1$, i.e., $t = i$, we have $f(q_{(N+t-i)}, q^*_{(i-t)}) = f(\mathbf{q})$ and the manipulating process ends. Adding from (B1) to (Bt0), we obtain (BFR) below. If $c > 1$, we continue the manipulating process to yield

$$f(q_{(N+t-i)}, q^*_{(i-t)}) + f(q_{(N+t-2i)}, q^*_{(i)}, q_{(i-t)}) \leq f(\mathbf{q}) + f(q_{(N+t-2i)}, q^*_{(2i-t)}), \tag{Bt1}$$

$$\vdots$$

$$f(q_{(N+t-(k-1)i)}, q^*_{((k-1)i-t)}) + f(q_{(N+t-ki)}, q^*_{(i)}, q_{((k-1)i-t)}) \leq f(\mathbf{q}) + f(q_{(2t)}, q^*_{(N-2t)}), \tag{BtK-1}$$

which follows

$$f(q_{(2t)}, q^*_{(N-2t)}) + f(q^*_{(2t)}, q_{(N-i)}, q^*_{(i-2t)}) \leq f(\mathbf{q}^*) + f(q_{(N+2t-i)}, q^*_{(i-2t)}), \tag{B2t1}$$

because $N + t - ki = 2t$. If $c = 2$, i.e., $i = 2t$, we have $f(q_{(N+2t-i)}, q^*_{(i-2t)}) = f(\mathbf{q})$. Thus, the manipulating process ends and we obtain (BFR) below by adding from (B1) to (B2t1). If $i > 2t$, i.e., $c > 2$, we will repeat the similar process as from (Bt1) to (BtK-1) until we obtain

$$\begin{aligned} &f(q_{(N+(c-1)t-(k-1)i)}, q^*_{((k-1)i-(c-1)t)}) + f(q_{(N+(c-1)t-ki)}, q^*_{(i)}, q_{((k-1)i-(c-1)t)}) \\ &\leq f(\mathbf{q}) + f(q_{(ct)}, q^*_{(N-ct)}). \end{aligned} \tag{BctK}$$

Since $i = ct$, we have

$$f(q_{(ct)}, q^*_{(N-ct)}) + f(q^*_{(i)}, q_{(N-i)}) \leq f(\mathbf{q}) + f(\mathbf{q}^*).$$

Adding from (B1) to (BctK), we obtain

$$\begin{aligned} f(q_{(N-i)}, q^*_{(i)}) \leq{} & [c(k-1)+1]f(\mathbf{q}) + cf(\mathbf{q}^*) \\ & - \sum_{m=0}^{c-1}\sum_{l=2}^{k} f(q_{(N+mt-li)}, q^*_{(i)}, q_{((l-1)i-mt)}) - \sum_{m=1}^{c} f(q^*_{(mt)}, q_{(N-i)}, q^*_{(i-mt)}). \end{aligned} \tag{BFR}$$

Multiplying t to both side of (BFR) and then adding $(N-t)f(q_{(N-i)}, q^*_{(i)})$ to both side to yield

$$\begin{aligned} Nf(q_{(N-i)}, q^*_{(i)}) \leq{} & (N-i)f(\mathbf{q}) + i \cdot f(\mathbf{q}^*) + (N-t)f(q_{(N-i)}, q^*_{(i)}) \\ & - t\sum_{m=0}^{c-1}\sum_{l=2}^{k} f(q_{(N+mt-li)}, q^*_{(i)}, q_{((l-1)i-mt)}) - t\sum_{m=1}^{c} f(q^*_{(mt)}, q_{(N-i)}, q^*_{(i-mt)}), \end{aligned}$$

where $N - i = [c(k-1)+1]t$, $N - t = ckt$, and $f(q^*_{(ct)}, q_{(N-i)}, q^*_{(i-ct)}) = f(q^*_{(i)}, q_{(N-i)})$. Note that the number of negative terms is $c(k-1)t + ct$, which is equal to kct and $N - t$. Thus, we obtain $\varphi(i,t)$ as shown by (1).

(2) We have the same process as case (1) until (BtK-1). To make the proof clear, we rewrite (BtK-1) as

$$f(q_{(N+t-(k-1)i)}, q^*_{((k-1)i-t)}) + f(q_{(N+t-ki)}, q^*_{(i)}, q_{((k-1)i-t)}) \leq f(\mathbf{q}) + f(q_{(2t)}, q^*_{(N-2t)}).$$

It follows $c_1 - 1$ times the similar process from (Bt0) to (BtK-1) with each time t increased from jt to $(j+1)t$, $j = 2, \ldots, c_1 - 1$ to obtain

$$\begin{aligned} & f(q_{(N+(c_1-1)t-(k-1)i)}, q^*_{((k-1)i-t)}) + f(q_{(N+(c_1-1)t-ki)}, q^*_{(i)}, q_{((k-1)i-t)}) \\ & \leq f(\mathbf{q}) + f(q_{(c_1t)}, q^*_{(N-c_1t)}). \end{aligned}$$

Since $i > c_1 t$, it follows

$$f(q_{(c_1t)}, q^*_{(N-c_1t)}) + f(q^*_{(c_1t)}, q_{(N-i)}, q^*_{(i-c_1t)}) \leq f(\mathbf{q}^*) + f(q_{(N+c_1t-i)}, q^*_{(i-c_1t)}), \tag{BC0}$$

$$f(q_{(N+c_1t-i)}, q^*_{(i-c_1t)}) + f(q_{(N+c_1t-2i)}, q^*_{(i)}, q_{(i-c_1t)}) \leq f(\mathbf{q}) + f(q_{(N+c_1t-2i)}, q^*_{(2i-c_1t)}), \tag{BC1}$$

$$f(q_{(N+c_1t-2i)}, q^*_{(2i-c_1t)}) + f(q_{(N+c_1t-3i)}, q^*_{(i)}, q_{(2i-c_1t)}) \leq f(\mathbf{q}) + f(q_{(N+c_1t-3i)}, q^*_{(3i-c_1t)}), \tag{BC2}$$

$$f(q_{(N+c_1t-3i)}, q^*_{(3i-c_1t)}) + f(q_{(N+c_1t-4i)}, q^*_{(i)}, q_{(3i-c_1t)}) \leq f(\mathbf{q}) + f(q_{(N+c_1t-4i)}, q^*_{(4i-c_1t)}), \tag{BC3}$$

$$\vdots$$

$$f(q_{(N+c_1t-(k-1)i)}, q^*_{((k-1)i-c_1t)}) + f(q_{(N+c_1t-ki)}, q^*_{(i)}, q_{((k-1)i-c_1t)}) \leq f(\mathbf{q}) + f(q_{((c_1+1)t)}, q^*_{(N-(c_1+1)t)}). \tag{BCK-1}$$

Since $i < (c_1+1)t$, (BCK-1) follows

$$\begin{aligned} &f(q_{((c_1+1)t)}, q^*_{(N-(c_1+1)t)}) + f(q_{((c_1+1)t-i)}, q^*_{(i)}, q_{(N-(c_1+1)t)}) \\ &\leq f(\mathbf{q}) + f(q_{((c_1+1)t-i)}, q^*_{(N-(c_1+1)t+i)}). \end{aligned} \tag{BD1}$$

It follows two cases:

(C1) if $(c_1+1)t - i = i$, i.e., $c_2 \equiv \left[\frac{2i}{t}\right] = c_1 + 1$; we have

$$f(q_{((c_1+1)t-i)}, q^*_{(N-(c_1+1)t+i)}) + f(q^*_{(i)}, q_{(N-i)}) \leq f(\mathbf{q}) + f(\mathbf{q}^*).$$

Thus, the manipulating process ends and we add from (B1) to (BD1) to obtain (BFR2) below.

(C2) if $(c_1+1)t - i < i$, i.e., $c_2 \geq c_1 + 1$. Note that since $c_1 \geq 1$ and $c_2 \geq 2c_1$, it is impossible to have $(c_1+1)t - i > i$, which requires that $c_1 + 1 > c_2$. Thus, (BD1) follows

$$\begin{aligned} &f(q_{((c_1+1)t-i)}, q^*_{(N-(c_1+1)t+i)}) + f(q^*_{(c_1+1)t-i}, q_{(N-i)}, q^*_{(2i-(c_1+1)t)}) \\ &\leq f(\mathbf{q}) + f(q_{(N+(c_1+1)t-2i)}, q^*_{(2i-(c_1+1)t)}). \end{aligned}$$

The right-hand of this inequality is similar to that of (BC1) except the parameter before

t is (c_1+1) instead of c_1 in (BC1). Thus, it follows the similar process from (BC2) to (BCK-1) with t increased from $(c_1+1)t$ to $(c_1+2)t$, i.e.,

$$\begin{aligned} &f(q_{(N+(c_1+1)t-(k-1)i)}, q^*_{((k-1)i-(c_1+1)t)}) + f(q_{(N+(c_1+1)t-ki)}, q^*_{(i)}, q_{((k-1)i-(c_1+1)t)}) \\ &\le f(\mathbf{q}) + f(q_{((c_1+2)t)}, q^*_{(N-(c_1+2)t)}). \end{aligned} \tag{BD2}$$

The similar manipulating process from (BD1) to (BD2) is repeated until we have $(c_1+f_1)t-i \ge i$ (f_1 is an integer) with each time t increased from $(c_1+j)t$ to $(c_1+j+1)t$, $j=2,\ldots,f_1-1$, to obtain

$$\begin{aligned} &f(q_{(N+(c_1+f_1-1)t-(k-1)i)}, q^*_{((k-1)i-(c_1+f_1)t)}) + f(q_{(N+(c_1+f_1-1)t-ki)}, q^*_{(i)}, q_{((k-1)i-(c_1+f_1-1)t)}) \\ &\le f(\mathbf{q}) + f(q_{(c_1+f_1)t)}, q^*_{(N-(c_1+f_1)t)}). \end{aligned} \tag{BEF1}$$

(BEF1) follows

$$\begin{aligned} &f(q_{(c_1+f_1)t)}, q^*_{(N-(c_1+f_1)t)}) + f(q_{((c_1+f_1)t-i)}, q^*_{(i)}, q_{(N-(c_1+f_1)t)}) \\ &\le f(\mathbf{q}) + f(q_{((c_1+f_1)t-i)}, q^*_{(N-(c_1+f_1)t+i)}). \end{aligned} \tag{BEFt1}$$

If $(c_1+f_1)t-i=i$, i.e., $c_2=c_1+f_1$, we have

$$f(q_{((c_1+f_1)t-i)}, q^*_{(N-(c_1+f_1)t+i)}) + f(q^*_{(i)}, q_{(N-i)}) \le f(\mathbf{q}) + f(\mathbf{q}^*).$$

Thus the manipulating process ends and we summarize the above manipulating process to obtain (BFR2). If $(c_1+f_1)t-i>i$, then we have $f_1=c_2-c_1+1$, i.e., $c_2+1=c_1+f_1$. (EEFt1) follows

$$\begin{aligned} &f(q_{((c_2+1)t-i)}, q^*_{(N-(c_2+1)t+i)}) + f(q_{((c_2+1)t-2i)}, q^*_{(i)}, q_{(N-(c_2+1)t+i)}) \\ &\le f(\mathbf{q}) + f(q_{((c_2+1)t-2i)}, q^*_{(N-(c_2+1)t+2i)}). \end{aligned}$$

Note that it is impossible to have $(c_2+1)t-2i>i$, which requires that $c_2+1>c_3$. Again, it follows two possibilities:

(C21) if $(c_1+f_1)t-2i=i$, i.e., $c_3=c_1+f_1=c_2+1$, we have

$$f(q_{(c_3t-2i)}, q^*_{(N-c_3t+2i)}) + f(q^*_{(i)}, q_{(N-i)}) \le f(\mathbf{q}) + f(\mathbf{q}^*).$$

The manipulating process ends and we obtain (BFR2).

(C22) If $(c_2+1)t-2i<i$, i.e., $c_3>c_2+1$, we have

$$\begin{aligned}&f(q_{((c_2+1)t-2i)},q^*_{(N-(c_2+1)t+i)})+f(q^*_{((c_2+1)t-2i)},q_{(N-i)},q^*_{(3i-(c_2+1)t)})\\&\leq f(\mathbf{q}^*)+f(q_{(N+(c_2+1)t-3i)},q^*_{(3i-(c_2+1)t)}).\end{aligned}$$

It follows the similar process from (BC3) to (BCK-1) with $(c_2+1)t$ replacing c_1t to obtain

$$\begin{aligned}&f(q_{(N+(c_2+1)t-(k-1)i)},q^*_{((k-1)i-(c_2+1)t)})+f(q_{(N+(c_2+1)t-ki)},q^*_{(i)},q_{((k-1)i-(c_2+1)t)}) \quad \text{(BEFt2)}\\&\leq f(\mathbf{q})+f(q_{(c_2+2)t},q^*_{N-(c_2+2)t}).\end{aligned}$$

Repeating the similar process from (BEFt1) to (BEFt2) with each time t increased from $(c_2+j)t$ to $(c_2+j+1)t$, $j=2,\ldots,f_2-1$ (f_2 is an integer), we obtain

$$\begin{aligned}&f(q_{(N+(c_2+f_2-1)t-(k-1)i)},q^*_{((k-1)i-(c_2+f_2-1)t)})+f(q_{(N+(c_2+f_2-1)t-ki)},q^*_{(i)},q_{((k-1)i-(c_2+f_2-1)t)})\\&\hfill \text{(BEF2)}\\&\leq f(\mathbf{q})+f(q_{(c_2+f_2)t},q^*_{N-(c_2+f_2)t}),\end{aligned}$$

where $(c_2+f_2)t-2i\geq i$, i.e., $c_2+f_2\geq c_3$. If $(c_1+f_2)t-2i=i$, then $f_2\equiv c_3-c_2$; If $(c_1+f_2)t-2i>i$, we have $f_2\equiv c_3-c_2+1$. It follows the similar process from (BEF1) to (BEF2) repeatedly with each time t increased from $(c_j+f_j)t$ to $(c_{j+1}+f_{j+1})t$, and $f_j=c_{j+1}-c_j+1$, $j=2,\ldots,s-2$, $s=min\{\ x\mid [\frac{xi}{t}]=\frac{xi}{t}\}$, to obtain

$$\begin{aligned}&f(q_{((N+(c_{s-1}+f_{s-1}-1)t-(k-1)i))},q^*_{((k-1)i-(c_{s-1}+f_{s-1}-1)t)})+f(q_{((c_{s-1}+f_{s-1})t)},q^*_{(i)},q_{((k-1)i-(c_{s-1}+f_{s-1}-1)t)})\\&\leq f(\mathbf{q})+f(q_{(c_{s-1}+f_{s-1})t},q^*_{N-(c_{s-1}+f_{s-1})t}).\end{aligned}$$

Note that $(c_{s-1}+f_{s-1})t-(s-1)i=i$ and $f_{s-1}=c_s-c_{s-1}$. It follows

$$\begin{aligned}f(q_{(c_st)},q^*_{N-c_st})+f(q_{(c_st-i)},q^*_{(i)},q_{(N-c_st)})&\leq f(\mathbf{q})+f(q_{(c_st-i)},q^*_{(N-c_st+i)}),\\&\vdots\\f(q_{(c_st-(s-1)i)},q^*_{(N-c_st+(s-1)i)})+f(q^*_{(i)},q_{(N-i)})&\leq f(\mathbf{q})+f(\mathbf{q}^*).\end{aligned}$$

Note that at least when $x=t$, we have $\left[\frac{xi}{t}\right]=\frac{xi}{t}$. In other words, s and c_s must exist. We summarize the above process to obtain

$$f(q_{(N-i)},q^*_{(i)}) \leq [c_s(k-1)+s]f(\mathbf{q})+c_sf(\mathbf{q}^*)-\sum_{m=0}^{c_1}\sum_{l=2}^{k}f(q_{(N+mt-li)},q^*_{(i)},q_{((l-1)i-mt)}) \tag{BFR2}$$

$$-\sum_{j=1}^{s-2}\sum_{m=c_j+1}^{c_{j+1}}\sum_{l=2+j}^{k}f(q_{((N+mt-li)},q^*_{(i)},q_{((l-1)i-mt)})$$

$$-\sum_{j=1}^{s}\sum_{m=c_{j-1}+1}^{c_j}f(q^*_{(mt-(j-1)i)},q_{(N-i)},q^*_{(ji-mt)})$$

$$-\sum_{m=c_{x-1}+1}^{c_s-1}\sum_{l=x+1}^{k}f(q_{(N+mt-li)},q^*_{(i)},q_{((l-1)i-mt)})$$

$$-\sum_{j=1}^{s-1}\sum_{m=c_j+1}^{c_s}f(q_{(mt-ji)},q^*_{(i)},q_{(N-mt+(j-1)i)}).$$

We multiply $\frac{t}{s}$ to both sides of (BFR2) and then add $(N-\frac{t}{s})f(q_{N-i},q_i^*)$ to both sides to yield $\varphi(i,t)$ as shown by (2). Since $s=min\{\ x\mid [\frac{xi}{t}]=\frac{xi}{t}\}$ and $c_s=\frac{si}{t}$, we have $i=\frac{c_s}{s}t$. Hence, $N-\frac{t}{s}=ki+t-\frac{t}{s}=\frac{t}{s}(c_sk+s-1)$. Note that $f(q^*_{(c_st-(s-1)i)},q_{(N-i)},q^*_{(si-c_st)})=f(q^*_{(i)},q_{(N-i)})$. The number of negative terms on the RHS of the above inequality is equal to

$$\begin{aligned}T&=\frac{t}{s}[(c_1+1)(k-1)+\sum_{j=1}^{s-2}(c_{j+1}-c_j)(k-1-j)\\&+\sum_{j=1}^{s}(c_j-c_{j-1})+(c_s-c_{s-1}-1)(k-s)+\sum_{j=1}^{s-1}(c_s-c_j)]\\&=\frac{t}{s}(c_sk+s-1),\end{aligned}$$

where $\sum_{j=1}^{s-2}(c_{j+1}-c_j)(k-1-j)=(c_2-c_1)(k-2)+(c_3-c_2)(k-3)+\cdots+(c_{s-1}-c_{s-2})(k+1-s)=-c_1(k-2)+\sum_{j=2}^{s-2}c_j+c_{s-1}(k+1-s)$, $\sum_{j=1}^{s}(c_j-c_{j-1})=c_s$, and $\sum_{j=1}^{s-1}(c_s-c_{j-1})=(s-1)c_s-\sum_{j=1}^{s-1}c_{j-1}$.

Appendix C

We will prove that if $f_i(\cdot)$ strictly decreases in i, i.e., $f_1(\cdot) > \cdots > f_N(\cdot)$, then $I(1) \subset I(2) \subset \cdots \subset I(N-1)$.

Proof: When only one task is offshored, clearly the least sensitive task, the Nth task will be offshored. That is, $I(1) = \{N\}$. First, we will prove that the Nth task must be included in $I(2)$. In order to prove this, we assume that the Nth task is not included in $I(2)$ when two tasks offshored. Without loss of generality, we suppose that the combination of the ith and jth, $i \neq j \neq N$, leads to the maximum output among all possible offshoring patterns when two tasks are offshored. Thus, we have $f(\mathbf{q}_{-ij}, q^*, q^*) > f(\mathbf{q}_{-iN}, q^*, q^*)$. It follows that

$$f(\mathbf{q}_{-i}, q^*) - f(\mathbf{q}_{-ij}, q^*, q^*) + f(\mathbf{q}_{-iN}, q^*, q^*) - f(\mathbf{q}_{-i}, q^*) < 0,$$

which is equivalent to

$$\int_{q^*}^{q} [f_j(\mathbf{q}_{-ij}, q^*, u) - f_N(\mathbf{q}_{-iN}, q^*, u)]du < 0$$

This contract to the assumption that $f_j(\cdot) > f_N(\cdot)$. Therefore, the Nth task must be included in $I(2)$, i.e., $I(1) \subset I(2)$.

Next, we prove the $N-1$th task must be in $I(2)$, that is, $f(\mathbf{q}_{-kN}, q^*, q^*) < f(\mathbf{q}_{-(N-1)N}, q^*, q^*)$, for $k \neq N-1$, because the Nth task is in $I(2)$. We use the contraction method again. Suppose that $f(\mathbf{q}_{-kN}, q^*, q^*) > f(\mathbf{q}_{-(N-1)N}, q^*, q^*)$. Then we have

$$f(\mathbf{q}_{-N}, q^*) - f(\mathbf{q}_{-kN}, q^*, q^*) + f(\mathbf{q}_{-(N-1)N}, q^*, q^*) - f(\mathbf{q}_{-N}, q^*) < 0,$$

which is equivalent to

$$\int_{q^*}^{q} [f_k(\mathbf{q}_{-kN}, q^*, u) - f_{N-1}(\mathbf{q}_{-N-1N}, q^*, u)]du < 0$$

This contract to the assumption that $f_k(\cdot) > f_{N-1}(\cdot)$. Therefore, the $N-1$th task must be included in $I(2)$. Similarly, we have $I(2) \subset I(3)$ and $N-2$th task must be in $I(3)$. This process continues till we have $I(1) \subset I(2) \subset \cdots \subset I(N-1)$ with $N-k$th task must be included in $I(k+1), k = 1, 2, \cdots, N-1$.

Appendix D

Proof: The conditions for task 3 to be offshored are

$$f(q, q, q^*) > f(q^*, q, q), \tag{D1a}$$

$$f(q, q, q^*) > f(q, q^*, q). \tag{D1b}$$

From (D1a), we have $f(q^*, q, q) - f(q, q, q) + f(q, q, q) - f(q, q, q^*) < 0$, which is equivalent to

$$\int_{q^*}^{q} f_3(q, q, u)du - \int_{q^*}^{q} f_1(u, q, q)du < 0. \tag{D2}$$

Similarly, from (D1b), we have $f(q, q^*, q) - f(q, q, q) + f(q, q, q) - f(q, q, q^*) < 0$, which is equivalent to

$$\int_{q^*}^{q} f_3(q, q, u)du - \int_{q^*}^{q} f_2(q, u, q)du < 0. \tag{D3}$$

When two tasks are offshored, offshoring task 1 and task 2 at the same time requires

$$f(q^*, q^*, q) > f(q^*, q, q^*), \tag{D4a}$$

$$f(q^*, q^*, q) > f(q, q^*, q^*). \tag{D4b}$$

From (D4a), we have $f(q^*, q^*, q) - f(q^*, q, q) + f(q^*, q, q) - f(q^*, q, q^*) > 0$, which is equivalent to

$$\int_{q^*}^{q} f_3(q^*, q, u)du - \int_{q^*}^{q} f_2(q^*, u, q)du > 0. \tag{D5}$$

Similarly, from (D4b), we have $f(q^*, q^*, q) - f(q, q^*, q) + f(q, q^*, q) - f(q, q^*, q^*) > 0$, which is equivalent to

$$\int_{q^*}^{q} f_3(q, q^*, u)du - \int_{q^*}^{q} f_1(u, q^*, q)du > 0. \tag{D6}$$

Let's define

$$G_1(t) \equiv \int_{q^*}^{q} f_3(q, t, u)du - \int_{q^*}^{q} f_1(u, t, q)du,$$

$$G_2(t) \equiv \int_{q^*}^{q} f_3(t, q, u)du - \int_{q^*}^{q} f_2(t, u, q)du.$$

(D2) and (D6) imply that $\frac{dG_1(t)}{dt} < 0$, which leads to $\int_{q^*}^{q} f_{23}(q, t, u)du - \int_{q^*}^{q} f_{12}(u, t, q)du < 0$. Similarly, (D3) and (D5) imply that $\frac{dG_2(t)}{dt} < 0$, which leads to $\int_{q^*}^{q} f_{13}(t, q, u)du - \int_{q^*}^{q} f_{12}(t, u, q)du < 0$. Therefore, $f_{12} > f_{13}$ and $f_{12} > f_{23}$ are necessary conditions.

Appendix E

Proof:

First, we prove that $f_{ij} > f_{ih}$ and $f_{ij} > f_{jh}$ are necessary conditions. When one task is offshored, the conditions for task h ($h \neq i$, $h \neq j$) to be offshored are

$$f(\mathbf{q}_{-h}, q^*) > f(\mathbf{q}_{-i}, q^*), \tag{E1}$$

$$f(\mathbf{q}_{-h}, q^*) > f(\mathbf{q}_{-j}, q^*). \tag{E2}$$

From (E1), we have $f(\mathbf{q}_{-h}, q^*) - f(\mathbf{q}) + f(\mathbf{q}) - f(\mathbf{q}_{-i}, q^*) > 0$, which is equivalent to

$$\int_{q^*}^{q} [f_h(\mathbf{q}_{-h}, u) - f_i(\mathbf{q}_{-i}, u)]du < 0.$$

Similarly, from (E2), we obtain

$$\int_{q^*}^{q} [f_h(\mathbf{q}_{-h}, u) - f_j(\mathbf{q}_{-j}, u)]du < 0.$$

Offshoring task i and task j at the same time requires $f(\mathbf{q}_{-ij}, q^*, q^*) - f(\mathbf{q}_{-ih}, q^*, q^*) > 0$ and $f(\mathbf{q}_{-ij}, q^*, q^*) - f(\mathbf{q}_{-hj}, q^*, q^*) > 0$. Following the similar logic of Appendix D, we can prove that $f_{ij} > f_{ih}$ and $f_{ij} > f_{jh}$ are necessary conditions.

It is clear that this result can be extended to the case that for any $k \neq i, j, h$, if $f(\mathbf{q}_{-k}, q^*) > f(\mathbf{q}_{-i}, q^*)$ or $f(\mathbf{q}_{-k}, q^*) > f(\mathbf{q}_{-j}, q^*)$ holds. Offshoring task i and task j at the same time requires $f(\mathbf{q}_{-ij}, q^*, q^*) - f(\mathbf{q}_{-ik}, q^*, q^*) > 0$ or $f(\mathbf{q}_{-ij}, q^*, q^*) - f(\mathbf{q}_{-kj}, q^*, q^*) > 0$, which leads to $f_{ij} > f_{ik}$ or $f_{ij} > f_{jk}$, following the same logic as in Appendix D.