# Offshoring and Job Polarisation between Firms*

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**Abstract**

We set up a general equilibrium model, in which offshoring to a low-wage country can lead to job polarisation in the high-wage country. Job polarisation is the result of a reallocation of labour across firms that differ in productivity and pay wages that are positively linked to their profits by a rent-sharing mechanism. Offshoring involves fixed and task-specific variable costs, and as a consequence it is chosen only by the most productive firms, and only for those tasks with the lowest variable offshoring costs. A reduction in those variable costs increases offshoring at the intensive and at the extensive margin, with domestic employment shifted from the newly offshoring firms in the middle of the productivity distribution to firms at the tails of this distribution, paying either very low or very high wages. We also study how the reallocation of labour across firms affects economy-wide unemployment. Offshoring reduces unemployment when it is confined to high-productivity firms, while this outcome is not guaranteed when offshoring is also chosen by low-productivity firms.

JEL-Classification: F12, F16, F23
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# 1 Introduction

The polarisation of jobs is a well-documented phenomenon for many industrialised economies.[1] To rationalise this observation, economists usually refer to the *routinisation hypothesis* (cf. Autor et al., 2003), which explains the disappearance of jobs from the middle of the wage distribution by a movement of workers from jobs concentrated on routine tasks to jobs highly intensive in abstract or service tasks at the upper and lower tail of the wage distribution, respectively. Whereas widespread access to data on tasks and occupations has made this hypothesis a particularly attractive avenue for empirical research, recent evidence from firm-level data is indicative of an *alternative* channel so far ignored in the literature: the movement of workers between firms that differ in their wage payments (Harrigan et al., 2016; Heyman, 2016; Kerr et al., 2016). Using information on the employment changes in a sample of 5,754 German establishments between 1999 and 2005, Figure 1 documents job polarisation between those establishments, with employment in establishments with initially high and low average wage rates increasing and employment in establishments paying intermediate wage rates decreasing.[2]


*Source*: LIAB dataset from the Institute for Employment Research in Nuremberg. Dots capture employment changes for 20 establishment groups (of equal size); The solid line depicts the outcome of local polynominal smoothing, using Epanechnikov kernel-weighting with degree 1 and bandwidth 1.71.

**Figure 1:** *Job polarisation between German establishments*

[1]For instance, Autor et al. (2006, 2008) and Autor and Dorn (2013) provide evidence for the US, whereas Dustmann et al. (2009) and Goos et al. (2009, 2014) document job polarisation for various European economies.

[2]For constructing Figure 1, we only consider establishments that were observed in the base and in the end year and, to control for the far-reaching labour market reforms in Germany between 2003 and 2005 as well as for other macroeconomic shocks, we have normalised the data by expressing employment changes relative to the overall trend. The reallocation of workers shown in Figure 1 is robust to changes in the covered time span and looks similar for the periods 1999-2003 or 1999-2007. The pattern is also robust to changes in the number of wage groups, with a u-shape being also found when accounting for 10 or 100 establishment categories, and it remains unaffected if managers are excluded from the sample of workers.

We argue in this paper that job polarisation along the firm margin, as captured by Figure 1, can be explained by the deepening of international economic integration in the recent past. In particular, relying on the two well-established facts that (i) more productive firms pay higher wages than their less productive competitors and that (ii) there is selection of the most productive firms into international markets, we show that job polarisation can be explained by the increased offshoring of production from high- to low-wage countries.[3] The story we have in mind to explain job polarisation between firms is straightforward: Suppose offshoring has fixed and variable costs, where the latter are task-specific as in Grossman and Rossi-Hansberg (2008). Furthermore, firms differ in their productivity, and – crucially – there is some mechanism linking firm productivity to firm-level average wages. In equilibrium, high-wage firms offshore some of their tasks, while low-wage firms do not offshore at all. It is now natural for an across-the-board decrease in variable offshoring costs to lead to job polarisation: Newly offshoring firms, which pay intermediate wages, since they are less productive than the incumbent offshoring firms, but more productive than the non-offshoring firms, reduce the number of their domestic jobs. This reduction in labour demand causes a downward pressure on wages, and therefore incumbent offshoring firms as well as purely domestic non-offshoring firms – i.e. the firms paying either very high or very low wages – increase their domestic employment.[4]

To formalise the story outlined above and to shed light on its possible limitations, we set up a general equilibrium model with monopolistic competition among heterogeneous firms. As in Lucas (1978) each firm is run by an entrepreneur, who hires workers for production. Agents are equally productive as workers, but differ in terms of their entrepreneurial ability, which is instrumental for the productivity of firms and thus for the profit income that accrues to the entrepreneur as residual claimant. The entry of firms and the economy-wide supply of labour are then jointly determined by the decentralised occupational choice of agents, with entrepreneurial ability being pivotal for the decision of who becomes an entrepreneur and who becomes a worker. To analyse offshoring decisions, we place our analysis in an asymmetric two-country model, in which firms from an industrialised North have an incentive to offshore the production of tasks to a developing South, with lower wages. Trade in tasks is associated with fixed and variable offshoring costs. Assuming that offshoring costs are the same for all producers, there is selection of firms into offshoring by

[3] Supportive evidence for the first fact can be found in empirical labour market research (see Blanchflower et al. 1996 and Abowd et al. 1999 for two early contributions and Card et al. 2013 for a more recent one). Regarding the second fact, Bernard and Jensen (1995) provided first systematic evidence that exporters are larger and more profitable than non-exporters, and the subsequent literature has pointed out that these differences are due to a selection of better firms into exporting (see Bernard and Jensen, 1999). Recent work by Hummels et al. (2014) and Moser et al. (2015) shows that the patterns of international market participation are similar if one considers offshoring instead of exporting.

[4] Using offshoring information for 1999, we find that the u-shape in Figure 1 vanishes when restricting the sample to establishments that were already classified as offshorers in the first observation year. This suggests that the asymmetric behavior of offshoring and non-offshoring firms may well be an important determinant of the job polarisation depicted in Figure 1.

productivity, establishing an (endogenous) *extensive firm margin* that separates highly productive offshoring firms from less productive non-offshoring firms. Assuming that variable offshoring costs are task-specific as in Grossman and Rossi-Hansberg (2008), there is, in addition, an (endogenous) *extensive task margin* that separates tasks kept onshore from tasks put offshore.

A framework for the analysis of job polarisation requires, of course, that not all workers are paid the same wage. We therefore augment our model of offshoring by a labour market model featuring rent sharing due to a fair-wage-effort mechanism (cf. Akerlof and Yellen, 1990; Egger and Kreickemeier, 2012; Amiti and Davis, 2012). The resulting framework generates a positive link between firm success and wages, because in the presence of fairness preferences firms with higher operating profits have to pay higher wages in order to elicit the full level of non-contractible effort from their workforce. We associate offshoring with vertical multinationals and, in line with evidence from Budd and Slaughter (2004) and Budd et al. (2005), assume that intra-firm rent sharing exists within and across borders so that wages of both domestic and foreign workers are positively correlated with firm-level profits. The equilibrium then features two continuous wage distributions, one in each country, with some workers in each country being unemployed due to the labour market imperfection. Whereas the specific motive for rent sharing considered here is attractive from the perspective of analytical tractability, relying on a fair-wage mechanism is by no means essential for our results.[5]

To see how in the model outlined above job polarisation is linked to the phenomenon of offshoring, it is key to understand the firm-level employment effects of offshoring in the source country. For this purpose, we distinguish between a *direct effect*, which captures the domestic firm-level response to lower offshoring costs either by firms previously producing all their tasks onshore (new offshorers) or by firms expanding the share of tasks put offshore (incumbent offshorers), and an *indirect effect*, which captures the domestic reallocation of workers changing their employment status in response to the direct effect. As discussed in Egger et al. (2015), there are two countervailing forces that contribute to the direct effect: On the one hand, offshoring firms replace domestic workers by foreign ones in those tasks newly put offshore (the *relocation effect*). On the other hand, there is cost saving from offshoring, which makes the offshoring firm more competitive and leads to expansion of domestic employment in all tasks that remain onshore (the *productivity effect*). Similar to Grossman and Rossi-Hansberg (2008), the productivity effect works on infra-marginal tasks

[5]Any labour market model producing the result that more productive firms pay higher wages provides a vehicle for linking job polarisation to offshoring. A possible alternative to the labour market model considered here is the model of Helpman et al. (2010), which features random matching of heterogeneous firms à la Melitz (2003) and workers that have match-specific abilities. Firms can screen their applicants to increase the average ability of their workforce, and there is rent sharing due to Nash bargaining at the firm level. With a rent-sharing mechanism as in Helpman et al. (2010), the reallocation of workers due to offshoring would be similar to our model, and hence job polarisation can also materialise in a thus modified framework. Even if wage differentials were due to positive assortative matching between heterogeneous firms and heterogeneous workers as in Sampson (2014), the mechanism leading to job polarisation in our model remains valid.

only, and it therefore becomes more relevant as the share of tasks moved offshore increases. As a consequence, the relocation effect is dominant when only few firms offshore just a small fraction of their tasks due to high offshoring costs, whereas the productivity effect is likely to dominate when offshoring of many tasks becomes common practice among high- and low-productivity firms due to low offshoring costs.[6]

Most important for explaining job polarisation, incumbent and new offshorers are affected by the direct firm-level employment effects in different ways. Whereas incumbent offshoring firms marginally expand their set of offshored tasks if the cost of offshoring declines, newly offshoring firms shift abroad a discrete mass of tasks, including the marginal task as well as all infra-marginal ones. Hence, there exists an intermediate level of offshoring costs such that via the direct effect domestic employment of incumbent offshorers paying high wages increases while domestic employment of new offshorers paying intermediate wages falls. Provided that the direct employment effects in newly and incumbent offshoring firms add up to an overall job loss, domestic workers set free can be hired by other firms, including purely domestic ones paying low wages. Thereby our model is able to generate the inter-firm worker movements illustrated in Figure 1, with firms in the middle of the wage distribution losing jobs at the expense of firms at the opposite ends of the wage distribution.

Since our model features economy-wide unemployment as a consequence of rent sharing, we can also use it to address the politically charged question of whether offshoring to low-wage countries destroys jobs in high-wage countries in the aggregate.[7] We show that our model gives a nuanced answer to this question, the main reason being that in general equilibrium it is of course possible for workers who have lost their jobs due to offshoring to find employment elsewhere. Specifically, for these workers our model points to three possible alternatives to unemployment: they may be hired as production workers in a different firm, they may work in the offshoring service sector, or they may become entrepreneurs themselves.[8] Economy-wide unemployment in our model increases in the extent of wage dispersion between firms, ceteris paribus, since the latter is a measure for the severeness of the labour market distortion. We show that offshoring reduces wage dispersion in the North relative to autarky if the offshoring cost is very high, and therefore only the most productive firms use this option. With the labour market distortion less severe, offshoring leads to

[6]The existence of two countervailing forces in the determination of the direct firm-level employment effect and changes in their relative importance provide a rationale for the inconclusive empirical evidence on the consequences of offshoring for firm-level employment (cf. Sethupathy, 2013; Hummels et al., 2014; and Moser et al., 2015).

[7]As pointed out by The Economist (2009), "Americans became almost hysterical" about the job destruction due to offshoring, when Forrester Research predicted a decade ago that 3.3 million American jobs will be offshored until 2015.

[8]Evidence from matched worker-firm-owner data shows that individuals who are unemployed (cf. Berglann et al., 2011) or displaced from their job (cf. von Greiff, 2009) are more likely to become entrepreneurs. Furthermore, Autor et al. (2014), Artuç and McLaren (2015), and Keller and Utar (2015) provide evidence that the movement of workers to other sectors is an important channel for the adjustment of labour markets to globalisation shocks.

lower aggregate unemployment in this case. We also show that this positive labour market effect can be overturned at low levels of offshoring cost, since in this case wage dispersion between firms increases.

Despite the voluminous empirical evidence on job polarisation (see for example Autor et al., 2006; Goos and Manning, 2007; Goos et al., 2009), there is a surprisingly small number of papers providing theoretical reasoning for Autor et al.'s (2003) *routinisation hypothesis*. As a notable exception, Acemoglu and Autor (2011) set up a model featuring an endogenous assignment of skills to tasks. Distinguishing three different worker types, they show that the replacement of routine tasks through technological change or offshoring can explain job polarisation, with displaced medium-skilled workers being reallocated to tasks previously performed by better-paid high-skilled workers or worse-paid low-skilled workers, respectively. Costinot and Vogel (2010) formulate a model, in which a continuum of workers sorts across a continuum of tasks depending on their comparative advantage. As pointed out by the authors, this model is also equipped to explain job polarisation, for reasons not too different from those outlined by Acemoglu and Autor (2011). However, focussing on a model with atomistic firms, neither Costinot and Vogel (2010) nor Acemoglu and Autor (2011) can explain job polarisation along the firm dimension reported in Figure 1. Hence, our model complements existing theoretical work by highlighting a new channel through which job polarisation can materalise: the creation and destruction of jobs by firms that differ in their wage payments.

Our model also builds on a large literature that studies the consequences of offshoring to low-wage countries (cf. Jones and Kierzkowski, 1990; Feenstra and Hanson, 1996; Grossman and Rossi-Hansberg, 2008; Rodriguez-Clare, 2010; Acemoglu et al., 2015), including several recent contributions that place their analysis in a framework with heterogeneous firms (cf. Antràs and Helpman, 2004; Antràs et al., 2006; Davidson et al., 2008; Groizard et al., 2014; Egger et al., 2015). Furthermore, accounting for labour market imperfections, our paper is related to an old and well established literature dealing with the effects of globalisation on unemployment and wage inequality (cf. Brecher, 1974; Davidson et al., 1988; Hosios, 1990), and more directly to the recent papers by Egger and Kreickemeier (2009), Helpman et al. (2010), Amiti and Davis (2012), and Egger and Kreickemeier (2012), who analyse the link between international goods trade, residual wage inequality and unemployment in models with firm heterogeneity. Although the rationale for a wage premium in high-productivity firms in those papers extends one to one to our model, there is an important difference between international trade and offshoring regarding the induced reallocation of labour in the home country. Whereas international trade in these models unambiguously leads to a shift of labour away from the low-wage firms towards the high-wage firms, offshoring in our model has the potential to reallocate labour away from medium-wage firms towards high- *and* low-wage firms. Hence, explaining job polarisation along the firm dimension by international market integration requires that the market integration is associated with more offshoring.

The paper is structured as follows. Section 2 presents the main building blocks of our model and Section 3 introduces the rent-sharing mechanism. In Section 4, we solve for the general equilibrium. In Section 5, we analyse the effects of falling (variable) offshoring costs on the domestic employment of offshoring and non-offshoring firms and characterise the conditions under which job polarisation materialises. In Section 6, we determine the effects of offshoring on economy-wide unemployment. Section 7 concludes with a summary of the most important results.

## 2 The model: basics

We set up a two-country model of offshoring, in which a country called South, that in equilibrium has low wages, provides labour input for the offshore task production of firms that have their headquarters in a high-wage country, called North. North has an endogenous mass $M$ of monopolistically competitive firms that produce horizontally differentiated goods. Each firm faces a demand function

$$x(v) = Ap(v)^{-\sigma}, \tag{1}$$

where $A$ is a variable capturing market size, $v$ indexes the firm, and $\sigma > 1$ is the constant price elasticity of demand. Following Acemoglu and Autor (2011) we model firm $v$'s production as the mapping of a continuum of tasks, uniformly distributed over the unit interval, into an output good using a Cobb-Douglas technology. Firms use the same technology in all tasks, with one unit of labour required for the production of one unit of each task. Denoting the production and usage of task $\eta$ in firm $v$ by $q(v, \eta)$, we specify the production function as

$$q(v) = \varphi(v) \exp\left[\int_0^1 \ln q(v, \eta) d\eta\right], \tag{2}$$

where $\varphi(v)$ is a firm-specific productivity parameter. Tasks differ in terms of the variable costs for moving them offshore, and without loss of generality we order the tasks in such a way that these costs are increasing in $\eta$. Specifically, we borrow from Grossman and Rossi-Hansberg (2008, p. 1986) the functional form:

$$\hat{t}(\eta) = \tau(1-\eta)^{-t}, \tag{3}$$

with $\tau > 1$ and $t > 0$. Thereby, $\tau$ represents the variable offshoring cost for the task with the lowest such cost, and with the elasticity of $\hat{t}(\eta)$ given by $t\eta/(1-\eta)$, higher values of shape parameter $t$ are associated with a more steeply increasing offshoring cost schedule.

A profit maximising firm in our model has to make three decisions: It must decide whether or not to offshore part of its task production (this determines the *extensive firm margin*), and – in case it chooses to offshore at all – it needs to decide for which tasks it wants to use this option (this is the *extensive task margin*), and how much labour it wants to employ in the production of each task (this is the *intensive task margin*). We focus on the determination of the task margins first and postpone the determination of the extensive firm margin until Section 4.

The extensive task margin is determined by indifference condition $\hat{t}[\hat{\eta}(v)]w_s(v) = w_n(v)$, where $w_n(v)$ and $w_s(v)$ denote the wage rates paid by firm $v$ in the North and the South, respectively, and $\hat{\eta}(v)$ denotes both the marginal task offshored and the share of tasks offshored by firm $v$.[9] Substituting for $\hat{t}(\eta)$ from Eq. (3), we can rewrite the indifference condition for the extensive task margin as

$$\hat{\eta}(v) = 1 - \left[\frac{\tau w_s(v)}{w_n(v)}\right]^{\frac{1}{t}}, \tag{4}$$

with general equilibrium constraints introduced below ensuring $\hat{\eta}(v) \in (0,1)$.

Turning to the intensive task margin, the Cobb-Douglas technology in Eq. (2) implies that profit maximising firms allocate expenditure equally across tasks. We show in the Appendix that the resulting unit cost function can be written as

$$c(v) = \frac{w_n(v)}{\varphi(v) \exp\{I(v) \ln \kappa[\hat{\eta}(v)]\}}, \tag{5}$$

where $I(v)$ is an indicator function, taking the value of one in the case of offshoring and the value of zero otherwise, and

$$\kappa[\hat{\eta}(v)] \equiv \left(\frac{1}{[1-\hat{\eta}(v)]\exp[\hat{\eta}(v)]}\right)^t. \tag{6}$$

Thereby, $\kappa[\hat{\eta}(v)]$ is firm $v$'s productivity gain due to offshoring in the case where the firm has chosen both task margins in order to minimise unit costs. Intuitively, we have $\kappa[\hat{\eta}(v)] = 1$ if $\hat{\eta}(v) = 0$ and $d\kappa[\hat{\eta}(v)]/d\hat{\eta}(v) > 0$.[10]

# 3 The labour market

We now show how firm-specific wage rates $w_\ell(v)$ are determined in our model. In accordance with a large body of empirical labour market research, we postulate a framework that generates firm-specific wage rates due to firm-level rent-sharing. As in Egger and Kreickemeier (2012) and Egger et al. (2013), the particular rent sharing model we are using is based on the fair-wage effort

[9] In order to save on notation, we omit country indices $\ell = n, s$ whenever it does not cause confusion.

[10] The offshoring-induced increase in productivity at the firm level is the direct analogue in our framework to the economy-wide *productivity increase* in the model of Grossman and Rossi-Hansberg (2008).

mechanism of Akerlof and Yellen (1990).[11] We are following Akerlof and Yellen (1990) in assuming that workers exert full effort, normalised to one, if and only if they are paid at least the wage they consider fair, $\hat{w}_\ell$, $\ell = n, s$, while they reduce their effort proportionally if the wage falls short of the fair wage. The functional relationship between effort $e$ and wage rate $w_\ell$ is therefore given by

$$e = \min\left\{\frac{w_\ell}{\hat{w}_\ell}, 1\right\}. \tag{7}$$

Firms are wage setters, and as in other efficiency wage models they set $w_\ell$ in order to minimise the cost per efficiency unit of labour. Given Eq. (7), firms have no incentive to pay less than $\hat{w}_\ell$, since effort would fall proportionally with the wage. Paying more than $\hat{w}_\ell$ would increase the wage per efficiency unit of labour, and therefore firms would not do that either, unless it were necessary in order to attract the optimal number of workers. Attracting workers is clearly no issue for firms in the presence of involuntary unemployment, and we therefore have $w_\ell = \hat{w}_\ell$ whenever such unemployment exists in equilibrium.

Workers' subjective evaluation of a fair wage depends on a firm-internal and a firm-external reference point. Following Egger and Kreickemeier (2012) and Egger et al. (2013), we associate the firm-external reference point with the income a worker can expect outside the job, which equals the average labour income in the economy. Denoting the unemployment rate of production workers by $u_\ell$ and the average wage per production worker by $\bar{w}_\ell$, average labour income in country $\ell$ is given by $(1 - u_\ell)\bar{w}_\ell$. The firm-internal reference point is the operating profit of the firm, $\pi(v)$. Via this firm-specific reference point, rent sharing co-determines the fair wage in our model, and it therefore influences wage setting at the firm level. Assuming that the fair wage is a weighted geometric average of the two components we obtain

$$\hat{w}_\ell(v) = \pi(v)^\theta [(1 - u_\ell)\bar{w}_\ell]^{1-\theta}, \tag{8}$$

where $\theta \in (0, 1)$ captures the strength of the rent-sharing motive in the fair-wage considerations of workers. We show below that the equilibrium in our model features involuntary unemployment in both markets. Therefore, all firms, even those with low productivity, are able to attract workers by paying the fair wage, and hence we have $w_\ell(v) = \hat{w}_\ell(v)$, for $\ell = n, s$.[12] The specification of the

[11] The fair-wage-effort mechanism is supported by empirical evidence from laboratory experiments (cf. Fehr et al., 1993, 1998b,a), field experiments (cf. Gneezy and List, 2006; Kube et al., 2013), and natural experiments (cf. Lee and Rupp, 2007). See Charness and Kuhn (2011) as well as List and Rasul (2011) for surveys on laboratory and field experiments in labour economics. Fehr et al. (2009) provide a detailed survey over the extensive experimental evidence in favour of the fair-wage-effort mechanism.

[12] Since total firm-level profits are a determinant of firm-level wages in both countries, our model features not only national but also international rent-sharing. Evidence supportive of international rent sharing within multinational firms is provided by Budd and Slaughter (2004), Dobbelaere (2004), Budd et al. (2005), and Martins and Yang (2015). Egger and Kreickemeier (2013) develop a model with fair wage preferences as in Eq. (8) and show how international rent sharing can lead to a multinational wage premium in general equilibrium if countries are asymmetric.

fair wage constraint in Eq. (8) then implies that the intra-firm wage differential $w_s(v)/w_n(v)$ does not differ between firms:

$$\frac{w_s(v)}{w_n(v)} = \omega^{1-\theta} \qquad \text{with} \qquad \omega \equiv \frac{(1-u_s)\bar{w}_s}{(1-u_n)\bar{w}_n}, \tag{9}$$

where $\omega$ is the ratio of average labour incomes in the two countries. As a consequence, the extensive task margin $\hat{\eta}$ and the firm-level productivity gain due to offshoring $\kappa$ are the same for all firms:

$$\hat{\eta} = 1 - \tau^{\frac{1}{t}}\omega^{\frac{1-\theta}{t}}, \tag{4'}$$

$$\kappa = \left[\frac{1}{(1-\hat{\eta})\exp(\hat{\eta})}\right]^t. \tag{6'}$$

With $\kappa$ being not specific to the firm, it is now straightforward to derive the effect of offshoring on firm-level operating profits and firm-level wages. In doing so, we replace the generic firm index $v$ by firm productivity $\varphi$ and an index for the offshoring status of the firm ($o$ and $d$ for "offshoring" and "domestic", respectively). The relative operating profits and relative domestic wages of two firms with the same productivity but different offshoring status are jointly determined by

$$\frac{\pi_o(\varphi)}{\pi_d(\varphi)} = \left[\frac{w_o(\varphi)}{w_d(\varphi)}\right]^{\frac{1}{\theta}} \qquad \text{and} \qquad \frac{\pi_o(\varphi)}{\pi_d(\varphi)} = \left[\frac{w_o(\varphi)}{w_d(\varphi)}\frac{1}{\kappa}\right]^{1-\sigma},$$

where the first equation follows from the fair wage constraint and the second equation follows from goods market equilibrium (with $x = q$) and constant markup pricing. After simple transformations we get

$$\frac{\pi_o(\varphi)}{\pi_d(\varphi)} = \kappa^{\xi} \qquad \text{and} \qquad \frac{w_o(\varphi)}{w_d(\varphi)} = \kappa^{\theta\xi}, \tag{10}$$

where $\xi \equiv (\sigma-1)/[1+\theta(\sigma-1)] > 0$ is the constant *cross-regime* elasticity of firm-level operating profits with respect to the productivity gain from offshoring, $\kappa$. In direct analogy to Eq. (10), $\xi$ and $\theta\xi$ also denote the *within-regime* elasticities of firm-level operating profits and firm-level wages, respectively, with respect to firm-level productivity, $\varphi$. Hence, our model predicts, in line with a large empirical literature (cf. Blanchflower et al., 1996; Albaek et al., 1998; Idson and Oi, 1999; Lallemand et al., 2007; Frias et al., 2012), that more productive firms make higher profits and pay higher wages.

# 4 General equilibrium

Having derived links between the endogenous variables $\kappa$, $\hat{\eta}$ and the ratio of average labour incomes $\omega$, we now introduce general equilibrium constraints to fully solve the model and analyse its comparative static properties. All goods produced under monopolistic competition serve as inputs into a homogeneous consumption good that is produced under perfect competition with a CES technology, à la Ethier (1982): $Y = \left[\int_{v \in V} x(v)^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)}$, where $V$ denotes the set of available intermediates. We choose the consumption good as the *numéraire*, and therefore the market size variable $A$ in Eq. (1) is equal to $Y$, the total *volume* and *value* of the consumption good produced in the North. Trade is balanced, with North importing the task output produced in the South in exchange for the consumption good.

Following Lucas (1978), we assume that each monopolistically competitive firm is run by a single entrepreneur who acts as an owner-manager and is the residual claimant of the firm, receiving as remuneration the firm's operating profits net of fixed offshoring costs. Operating profits depend on the entrepreneurial ability of the owner-manager, which determines (and is equal to) the productivity of the firm, $\varphi$. We assume that the distribution of entrepreneurial ability is Pareto and has support on interval $[1, \infty)$: $G(\varphi) = 1 - \varphi^{-k}$, with shape parameter $k > \max\{\xi, 1\}$.[13] In the North, being an entrepreneur is one of three possible occupations. Individuals could alternatively seek employment as production workers (with uncertain wage and employment prospects), or they could work in the perfectly competitive offshoring service sector (earning a guaranteed wage $s$). In total, the North has an exogenous supply of $N_n$ risk-neutral individuals, each inelastically supplying one unit of labour. Differences in ability $\varphi$ only matter for the income of entrepreneurs and agents in the North choose the occupation yielding the highest expected income, acknowledging their realisation of $\varphi$. In the South, there are $N_s$ individuals, and we assume that they have no choice but to seek employment as production workers, which is the simplest possible way to ensure that the South is the low-wage country in equilibrium.

We now turn to the determination of the extensive firm margin $\chi$, the share of firms that choose offshoring rather than purely domestic production. Offshoring in our model involves a fixed cost $s$ resulting from using the services of one offshoring service worker. Two indifference conditions are crucial in the process of determining $\chi$. First, the marginal entrepreneur needs to be indifferent between career paths. We focus on interior equilibria with $\chi < 1$ (the parameter constraint required for this outcome is introduced below), and therefore the marginal entrepreneur runs a non-offshoring (domestic) firm, whose productivity we denote by $\varphi_d$. Assuming that the occupational choice is irreversible, for instance due to occupation-specific education, the indifference condition is given by

$$\pi_d(\varphi_d) = (1 - u_n)\,\bar{w}_n = s, \tag{11}$$

where incomes of all three career paths are endogenous. The second indifference condition requires

[13] The parameter constraint for $k$ guarantees finite positive values of all variables of interest.

the gain in operating profits for the marginal offshoring firm, whose productivity we denote by $\varphi_o$, to be equal to the offshoring fixed cost:

$$\pi_o(\varphi_o) - \pi_d(\varphi_o) = s. \tag{12}$$

Eqs. (11) and (12), together with the link between firm-level productivity and firm-level operating profits derived in the previous section, imply a relationship between $\kappa$ and the relative productivity $\varphi_d/\varphi_o$, and using $\chi = [1 - G(\varphi_o)]/[1 - G(\varphi_d)] = (\varphi_d/\varphi_o)^k$ we can write the equilibrium condition for the extensive firm margin as

$$\chi = \left(\kappa^{\xi} - 1\right)^{\frac{k}{\xi}}. \tag{13}$$

The last building block of general equilibrium in our model is the link between relative labour endowments of the two countries, $N_n/N_s$, and the ratio of average labour incomes, $\omega$. For the South, the equilibrium relationship between endowment $N_s$ and the average labour income is

$$(1 - u_s)\,\bar{w}_s = \gamma\frac{\sigma - 1}{\sigma}\frac{Y}{N_s}, \tag{14}$$

where $\gamma$ is the share of aggregate labour income accruing to the South, and we show in the Appendix that it can be written as

$$\gamma \equiv \frac{\hat{\eta}\chi(1 + \chi^{-\xi/k})}{1 + \chi}. \tag{15}$$

Intuitively, due to constant mark-up pricing the income of production workers is a fraction $(\sigma-1)/\sigma$ of total income, a fraction $\gamma$ of which accrues to workers in the South. With more offshoring at the extensive task and/or extensive firm margin, South's income share $\gamma$ rises, i.e. $\partial\gamma/\partial\hat{\eta} > 0$ and $\partial\gamma/\partial\chi > 0$. For the North, the analogous equation is given by

$$(1 - u_n)\,\bar{w}_n = (1 - \gamma)\,\frac{\sigma - 1}{\sigma}\frac{Y}{L}, \tag{16}$$

with the additional complication that the total supply of production workers in the North, denoted by $L$, is endogenous due to the occupational choice mechanism. In equilibrium $L$ and $N_n$ are linked by

$$L = \frac{(1 - \gamma)\,k(\sigma - 1)}{k - \xi + (1 - \gamma)\,k(\sigma - 1)}N_n, \tag{17}$$

which can be derived, as we show in the Appendix, by combining the resource constraint $N_n = L + (1 + \chi)M$ with the indifference condition in Eq. (11). It follows immediately from Eqs. (15) and (17) that more offshoring along the two extensive margins is accompanied by a reduction in

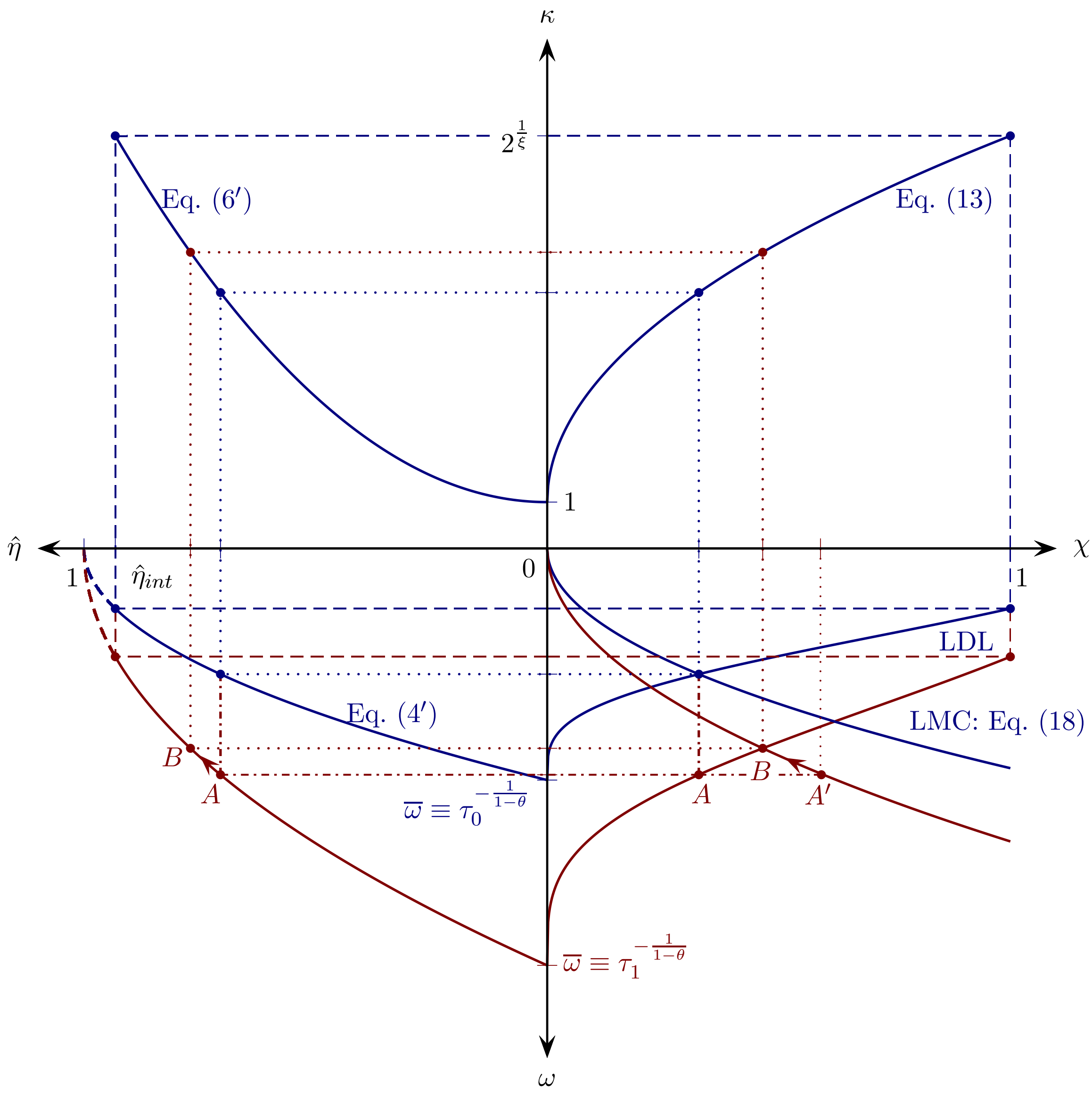


**Figure 2:** *General Equilibrium*

the supply of production workers.

Combining Eqs. (14), (16), and (17) we finally get

$$\omega = \frac{\gamma(\sigma-1)k}{k-\xi+(1-\gamma)\,k(\sigma-1)}\frac{N_n}{N_s}, \tag{18}$$

which we call the *labour market constraint* (LMC) since it links the three endogenous variables $\hat{\eta}$, $\chi$, and $\omega$ to the relative endowment $N_n/N_s$. For a constant relative labour endowment, Eqs. (15) and (18) imply that more offshoring along the two extensive margins leads to an increase in $\omega$, the relative average labour income of the South.

We can now solve for the four endogenous variables $\omega$, $\kappa$, $\hat{\eta}$ and $\chi$ using the indifference condition for the extensive task margin, Eq. (4′), the condition for the firm-level productivity gain resulting from the optimally chosen intensive task margin in Eq. (6′), the equilibrium condition for the extensive firm margin, Eq. (13), and the labour market constraint, Eq. (18). It is convenient to illustrate the determination of equilibrium in Figure 1. The graphical representations of Eqs. (13), (6′), and (4′) in Quadrants I to III are straightforward and do not require further elaboration. The upward sloping locus in Quadrant IV represents the labour market constraint from Eq. (18), drawn here as a functional relationship between $\chi$ and $\omega$, with $\hat{\eta}$ adjusting endogenously according to the indifference condition for the extensive task margin given in Quadrant III. The downward sloping locus in Quadrant IV is derived from the loci in Quadrants I to III, giving combinations of $\chi$ and $\omega$ that are compatible with indifference along the extensive firm margin and the extensive task margin as well as with the firm-level productivity gain resulting from the cost-minimising intensive task margin. Since it summarises demand conditions for labour in the two countries, we label the downward sloping locus in Quadrant IV the *labour demand locus* (LDL). Intuitively, offshoring is chosen by more firms if the relative average labour income of the South is lower and it therefore becomes relatively cheaper for firms to hire Southern labour.

Existence of an interior equilibrium requires that the two loci LMC and LDL intersect at $\chi < 1$. According to Eqs. (4′) and (18), a unique intersection point exists under the parameter constraint

$$\frac{\hat{\eta}_{int}(\sigma-1)k}{k-\xi+(1-\hat{\eta}_{int})k(\sigma-1)}\frac{N_n}{N_s} > (1-\hat{\eta}_{int})^{\frac{t}{1-\theta}}\,, \tag{19}$$

assumed in the following, where $\hat{\eta}_{int}$ is the maximum share of tasks that is offshored in an interior equilibrium with $\chi < 1$. Supremum $\hat{\eta}_{int}$ lies in the unit interval, and from Eqs. (6′) and (13) it is implicitly given by $2 = (1-\hat{\eta}_{int})^{-t\xi}\exp[-\hat{\eta}_{int}t\xi]$. Intuitively, condition (19) shows that an interior equilibrium requires the South to be sufficiently small, measured by its relative population size $N_s/N_n$. In this case, the wage differential between the two markets narrows quickly as more and more firms offshore more and more tasks, and hence the least productive firm always finds it advantageous to produce all tasks at home, thereby avoiding the fixed (and variable) costs of offshoring.

The effects of a reduction in variable offshoring costs $\tau$ can now be readily analysed using Figure **??**. A decline in $\tau$ from $\tau_0$ to $\tau_1$ implies that the indifference condition for the extensive task margin in Quadrant III is shifted downward: a given share of offshored tasks $\hat{\eta}$ is compatible with a higher average labour income in the South, ceteris paribus, if the offshoring cost is lower. The labour demand locus in Quadrant IV is shifted downward *to the exact same extent*, since, as shown above, profit maximisation implies that a constant extensive task margin $\hat{\eta}$ is accompanied by a constant external firm margin $\chi$. The change in $\omega$ compatible with a constant extensive task margin is indicated by point $A$ in Quadrants III and IV. The labour market constraint also shifts downward, but by less than the LDL. This is seen as follows: For a given $\hat{\eta}$, labour market equilibrium requires that an increase in $\omega$ is accompanied by an increase in $\chi$. Therefore, the shifted LMC in Figure ?? must go through a point such as $A'$ that is strictly to the right of point $A$. The new equilibrium is at point $B$, which satisfies both LMC and LDL. A reduction in the variable costs of offshoring therefore leads to more offshoring along the extensive firm and the extensive task margin. The firm-level productivity gain due to offshoring increases, while at the same time the increased relative demand for Southern workers leads to an increase in their relative average income.

## 5 Offshoring and firm level employment

We now turn to the effect of a change in offshoring costs on the allocation of workers between firms in the North. For this purpose, we first write domestic firm-level employment in non-offshoring firms and offshoring firms as

$$\ln l_d(\varphi, \hat{\eta}) = \ln(\sigma - 1) + (1-\theta)\xi \ln \varphi + \mu(\hat{\eta}), \tag{20}$$

$$\ln l_o(\varphi, \hat{\eta}) = \ln(\sigma - 1) + (1-\theta)\xi \ln \varphi + \mu(\hat{\eta}) + \lambda(\hat{\eta}), \tag{21}$$

where $\lambda(\hat{\eta})$ and $\mu(\hat{\eta})$ are defined as

$$\lambda(\hat{\eta}) \equiv [1 - t(1-\theta)\xi] \ln(1-\hat{\eta}) - t(1-\theta)\xi\hat{\eta}, \tag{22}$$

$$\mu(\hat{\eta}) \equiv -\left[\frac{(1-\theta)\xi}{k}\right] \ln \left\{1 + \chi(\hat{\eta}) + \frac{k(\sigma-1)}{k-\xi}\left[1 + (1-\hat{\eta})\chi(\hat{\eta}) - \hat{\eta}\chi(\hat{\eta})^{\frac{k-\xi}{k}}\right]\right\}. \tag{23}$$

Eqs. (20) and (21) show that domestic employment for both types of firms is increasing in firm productivity with constant elasticity $(1-\theta)\xi$. Furthermore, domestic firm-level employment is affected by the share of offshored tasks $\hat{\eta}$ via two separate channels, which we have summarised in the respective terms $\lambda(\hat{\eta})$ and $\mu(\hat{\eta})$: The term $\lambda(\hat{\eta})$ appears only in Eq. (21), indicating that it represents the direct link between $\hat{\eta}$ and employment in those firms that conduct offshoring. The term $\mu(\hat{\eta})$ is common to both equations, showing that it represents indirect market forces that affect employment in all firms equally. While Eqs. (20) and (21) are written as functions of the endogenous variable $\hat{\eta}$, we have shown above that there is a monotonic link between $\hat{\eta}$ and the exogenous offshoring cost parameter $\tau$. We can therefore analyse the comparative statics in terms of $\hat{\eta}$, in the understanding that an increase in $\hat{\eta}$ is the result of a decrease in $\tau$.

We focus on the direct effect of offshoring on domestic firm-level employment first, since the indirect effect common to all firms is a second-round effect that is easily understood once the sign of the first-round effect is known. As shown above, if an increase in $\hat{\eta}$ in our model is the result

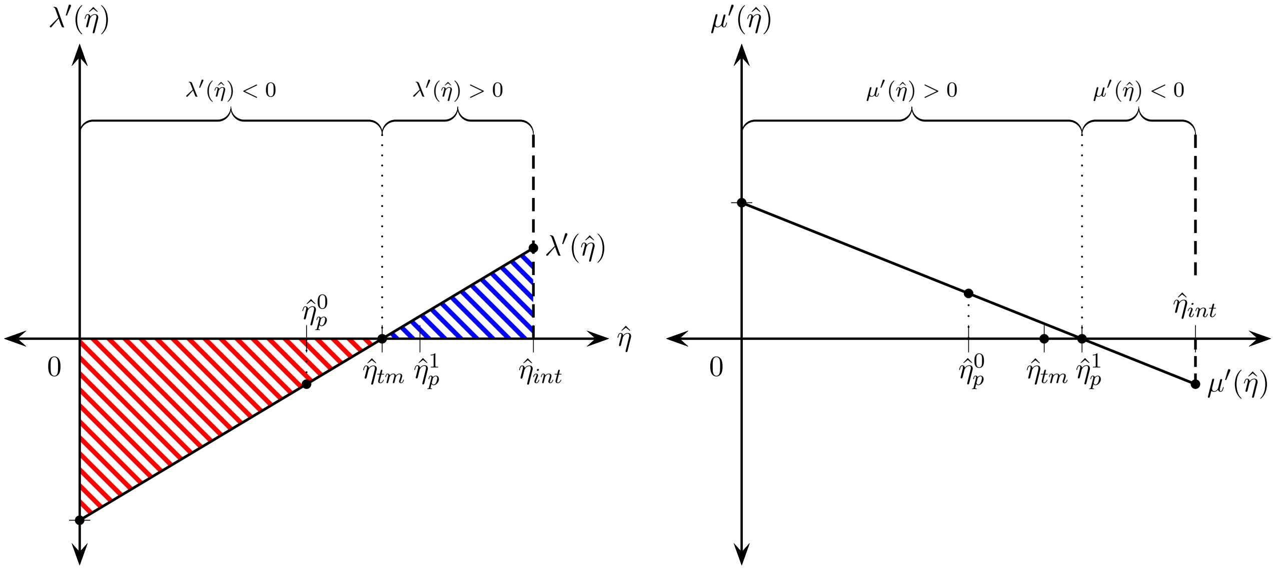


**Figure 3:** *Direct and indirect employment effects of offshoring*

of a reduction in $\tau$, then $\chi$, the share of firms that choose offshoring, increases as well. In the following we will therefore carefully distinguish the effect a reduction in $\tau$ has on the domestic employment in *incumbent offshoring firms* from the effect it has on domestic employment in the *newly offshoring firms*.

Eq. (21) shows that the direct effect on the log of domestic employment in incumbent offshoring firms is equal to $\lambda'(\hat\eta)$. It is the composite of a positive productivity effect, capturing the increase of domestic employment as a result of increased output, and a negative relocation effect, capturing the reduction in domestic employment due to the relocation of tasks to the South.[14] Since the productivity effect works on infra-marginal tasks only, as pointed out by Grossman and Rossi-Hansberg (2008), it is increasing in $\hat\eta$, therefore becoming more important at higher levels of offshoring. On the other hand, the absolute value of the relocation effect is decreasing in $\hat\eta$ since with a high $\hat\eta$ employment in newly offshored tasks is relatively low as firms have already shifted employment towards the cheaper tasks moved offshore before. Overall therefore, $\lambda'(\hat\eta)$ is negative at $\hat\eta = 0$, and it is monotonically increasing in $\hat\eta$, as illustrated in the left panel of Figure 3.

Whether or not $\lambda'(\hat\eta)$ turns positive at some $\hat\eta < \hat\eta_{int}$ depends on offshoring cost parameter $t$. As we show in the Appendix, there exists a threshold level $t_1$ that is implicitly defined by

$$\ln 2 = -t_1 \xi \ln\left(1 - \frac{1}{t_1(1-\theta)\xi}\right) - \frac{1}{1-\theta}, \tag{24}$$

and that has the following interpretation: If and only if $t > t_1$, $\lambda'(\hat\eta)$ becomes positive at some

[14]These two counteracting effects are also present in Grossman and Rossi-Hansberg (2008). However, the analogy is not perfect, since firms in the model of Grossman and Rossi-Hansberg are atomistic, and hence they frame their discussion in terms of changes in aggregate labour demand rather than in terms of changes in firm-level employment.

$\hat{\eta} < \hat{\eta}_{int}$, implying that the productivity effect dominates at high levels of $\hat{\eta}$, and therefore at this stage a reduction in variable offshoring costs increases domestic employment in incumbent offshoring firms. Intuitively, a high value of $t$ is conducive to $\lambda'(\hat{\eta}) > 0$ because the elasticity of the offshoring cost schedule is increasing in $t$ and therefore a high value of $t$ implies for a given $\hat{\eta}$ large cost savings on infra-marginal tasks, which in turn means that the productivity effect is large. In the following, we focus on the empirically relevant case $t > t_1$, which allows the domestic firm-level employment effect of offshoring to be negative (cf. Biscourp and Kramarz, 2007; Mion and Zhu, 2013; Hummels et al., 2014; Monarch et al., 2016) *or* positive (cf. Hakkala and Huttunen, 2015; Moser et al., 2015), depending on the share of offshored tasks $\hat{\eta}$. We denote by $\hat{\eta}_{tm}$, implicitly defined by $\lambda'(\hat{\eta}_{tm}) = 0$, the critical level above which $\lambda'(\hat{\eta})$ becomes positive.[15]

We now turn to the newly offshoring firms, and to the direct effect that an increase in $\hat{\eta}$ has on domestic employment in those firms. Newly offshoring firms move abroad in one go all tasks $\eta \leq \hat{\eta}$, and therefore the direct firm-level employment effect for those firms is given by $\lambda(\hat{\eta}) = \int_0^{\hat{\eta}} \lambda'(\eta) d\eta$. We depict $\lambda(\hat{\eta})$ in Figure 3 as the difference between the blue area below $\lambda'(\hat{\eta})$ and the red area above $\lambda'(\hat{\eta})$. Figure 3 shows that for values of $\hat{\eta}$ larger than but sufficiently close to $\hat{\eta}_{tm}$ an increase in $\hat{\eta}$ is associated with a direct domestic employment effect that is positive for incumbent offshoring firms ($\lambda'$ is positive) but negative for newly offshoring firms ($\lambda$ is negative). The latter effect is novel relative to Grossman and Rossi-Hansberg (2008): Since in their model the firm population is homogenous (firms are atomistic), the extensive firm margin is absent, and hence, using our terminology, all firms are *incumbent offshoring firms*. As we show in the following, the direct employment effect in newly offshoring firms, and the fact that it can be different from the effect in incumbent firms, is important for understanding the link between offshoring and job polarisation along the firm dimension.

The sum of the direct employment effects for the two groups of firms induces an indirect effect $\mu'(\hat{\eta})$, which represents second-round employment adjustments due to the changing market conditions and according to Eqs. (20) and (21) is the same for all firms. The fundamental logic behind the indirect effect is straightforward: A net employment loss in offshoring firms tends to put downward pressure on the wages of production workers. In response to lower wages all firms (including new entrants at the lower end of the productivity distribution) increase their domestic employment. The strength of the second-round effect $\mu'(\hat{\eta})$ is inversely related to the strength of the first-round employment response in offshoring firms, and it therefore becomes less positive as $\hat{\eta}$ increases. We show formally in the Appendix and illustrate in the right panel of Figure 3 that, consistent with this logic, $\mu'(\hat{\eta})$ is positive at $\hat{\eta} = 0$, and it becomes negative for high values of $\hat{\eta}$,

[15] Focussing on the domestic employment of incumbent offshoring firms in the U.S., Sethupathy (2013) does not find a significant effect of a reduction in the cost of offshoring to Mexico. In view of our model, this indicates an extensive task margin for US firms in the neighbourhood of $\hat{\eta}_{tm}$

provided that $\mu'(\hat{\eta}) = 0$ has a solution in interval $(0, \hat{\eta}_{int}]$.

This fundamental logic needs to be qualified, since workers losing their job due to a negative first-round effect can find employment in the offshoring service sector or become unemployed, thereby weakening the link between direct and indirect domestic firm-level employment effects. To ensure that these additional effects do not alter our results, we impose two parameter constraints, whose derivation is delegated to the Appendix. First, there is a minimum threshold for $t$, denoted by $t_2$ and implicitly defined by $\mu'(\hat{\eta}_{tm}) = 0$. A larger $t$ increases the elasticity of the variable offshoring cost schedule $\hat{t}(\eta)$ and thus weakens the extensive task margin, ceteris paribus. With fewer foreign workers hired by incumbent offshorers in response to a fall in $\tau$, relative foreign labour income increases by less and hence more firms start to offshore, thereby strengthening the extensive firm margin. This leads to a larger displacement of domestic workers by newly offshoring firms and thus provides a stronger impetus for the second-round reallocation of labour at $\hat{\eta}_{tm}$.

Second, we need to impose a constraint on parameters $\theta$ and $\sigma$ to ensure the existence of a *finite* threshold $t_2$ and assume that

$$1 - (\sigma - 1)\left[1 - \theta - \frac{\theta k}{k - \xi}\right] < 0. \tag{25}$$

For the limiting case of $\theta = 0$, labour markets are competitive and, in this case, (25) reduces to $\sigma > 2$. To understand this condition, it is worth noting that the labour requirement for the fixed cost of offshoring is the same for all firms. However, the displacement of workers by newly offshoring firms is larger ceteris paribus for firms with higher productivity, because these are the firms with larger domestic employment in each task prior to offshoring. For small new offshorers the displacement of domestic workers can therefore be dominated by the additional labour demand for the fixed cost of offshoring. Whereas a larger $t$ strengthens the displacement of workers by newly offshoring firms, the additional displacement by the marginal offshoring firm decreases, because it is a firm with lower productivity. Hence, for high values of $t$ it is possible that new offshorers actually increase their domestic labour demand, and to ensure that overall the labour demand of offshoring firms decreases, we must constrain the entry of small firms into offshoring. We can do so by introducing a lower threshold for $\sigma$, because a higher $\sigma$ increases the elasticity of profits with respect to productivity and hence makes it more difficult for smaller firms to bear the fixed cost of offshoring. This implies that $\mu'(\hat{\eta}_{tm}) > 0$ holds for all possible $t > t_2$.

In the case of $\theta > 0$, changes in $\sigma$ interact with the extent of rent sharing. This can be seen in Section 3, where we have shown that for $\theta > 0$ a larger $\sigma$ widens the wage dispersion. Since the wage dispersion is directly linked to the labour market distortion from rent sharing, we must impose a joint constraint on $\theta$ and $\sigma$ to make sure that a displacement of domestic workers by newly offshoring firms is neither offset by an increase in the labour demand for the fixed cost of offshoring nor offset by a strong increase in domestic unemployment due to an exacerbation of the labour market distortion.

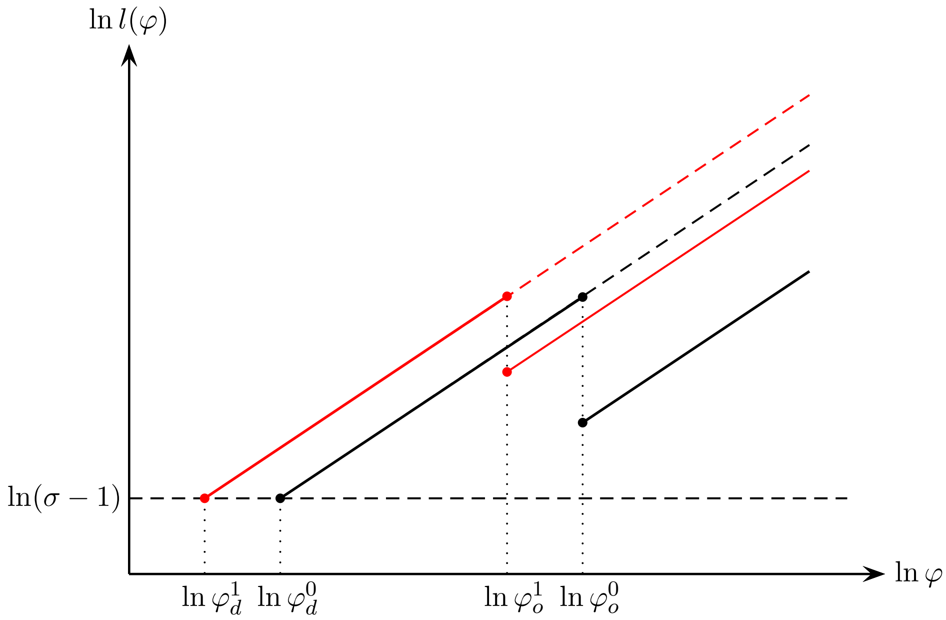


**Figure 4:** *The case of job polarisation*

**Proposition 1** *Let $\hat{\eta}_p^0$ be implicitly defined by $\lambda'(\hat{\eta}_p^0) + \mu'(\hat{\eta}_p^0) = 0$ and let $\hat{\eta}_p^1$ be equal to $\hat{\eta}_{int}$ if $\mu'(\hat{\eta}_{int}) > 0$ or implicitly defined by $\mu'(\hat{\eta}_p^1) = 0$ otherwise. Then, if $t > \max\{t_1, t_2\}$ and the parameter constraint in (25) holds, there exists an interval around $\hat{\eta}_{tm}$ with infimum $\hat{\eta}_p^0$ and supremum $\hat{\eta}_p^1$, such that a reduction of offshoring costs leads to job polarisation if $\hat{\eta} \in (\hat{\eta}_p^0, \hat{\eta}_p^1)$.*

**Proof** See the Appendix.

At the lower interval bound $\hat{\eta}_p^0$ employment in incumbent offshoring firms stays constant, as illustrated in Figure 3. Below this lower bound $\lambda'(\hat{\eta}) + \mu'(\hat{\eta})$ is negative, and therefore reducing $\tau$ would lead to less employment in (high-wage) incumbent offshoring firms, which is in contradiction to what is observed with job polarisation. At the upper interval bound $\hat{\eta}_p^1$ we have to distinguish the two possible cases $\hat{\eta}_p^1 < \hat{\eta}_{int}$ and $\hat{\eta}_p^1 = \hat{\eta}_{int}$. In the first case, employment in non-offshoring firms stays constant at $\hat{\eta} = \hat{\eta}_p^1$, and $\hat{\eta} > \hat{\eta}_p^1$ would imply $\mu'(\hat{\eta}) < 0$ and thus less employment in non-offshoring firms if $\tau$ falls. Again, this is in contradiction to what is observed with job polarisation. In the second case, $\hat{\eta} > \hat{\eta}_p^1$ is not possible. Inside the interval $\hat{\eta} \in (\hat{\eta}_p^0, \hat{\eta}_p^1)$ a reduction in $\tau$ leads to exactly the firm-level employment changes that is observed with job polarisation: lower employment in newly offshoring firms in the middle of the wage distribution and higher employment in non-offshoring firms as well as in incumbent offshoring firms at the two tails of the wage distribution.

Figure 4 is an alternative way of illustrating job polarisation in our model by showing firm-level domestic employment as a function of firm-level productivity. The black curve shows the initial equilibrium, with cutoff productivities denoted by $\varphi_d^0$ and $\varphi_o^0$, respectively. A small decline in $\tau$ leads to the red employment profile with cutoff productivities $\varphi_d^1$ and $\varphi_o^1$. Employment increases in non-offshoring firms with low productivity and in incumbent offshoring firms with high productivity, but decreases in newly offshoring firms, which have intermediate productivity levels.

## 6 Offshoring and Unemployment

As discussed above, the firm-specific fair wage constraints in our model are only binding if firms can attract the desired number of production workers in both countries by paying the fair wage implied by those constraints. This in turn requires that there is involuntary unemployment of production workers in both countries, since only in this case it is possible

1. for the marginal firm with productivity $\varphi_d$ to attract the desired number of workers, or indeed any workers, in the North by paying the domestic fair wage $\hat{w}_n(\varphi_d)$, and
2. for the marginal offshoring firm with productivity $\varphi_o$ to attract the desired number of workers, or indeed any workers, in the South by paying the foreign fair wage $\hat{w}_s(\varphi_o)$.

Here we show that our model necessarily features involuntary unemployment of production workers in both countries in any equilibrium that has more productive firms pay higher wages, implying that it is indeed an equilibrium that all active firms pay their workers the respective fair wage (but not more).

Crucial for this result are three conditions already introduced above. The first is the indifference condition for the marginal entrepreneur in the domestic economy $\pi_d(\varphi_d) = (1-u_n)\bar{w}_n$, the other two are the domestic fair wage constraint for the marginal firm with productivity $\varphi_d$ and the foreign fair wage constraint for the marginal offshoring firm with productivity $\varphi_o$:

$$\hat{w}_n(\varphi_d) = [\pi_d(\varphi_d)]^\theta [(1-u_n)\bar{w}_n]^{1-\theta}, \qquad \hat{w}_s(\varphi_o) = [\pi_o(\varphi_o)]^\theta [(1-u_s)\bar{w}_s]^{1-\theta}.$$

Together, these conditions imply

$$\hat{w}_n(\varphi_d) = (1-u_n)\bar{w}_n \qquad \text{and} \qquad \hat{w}_s(\varphi_o) = D(1-u_s)\bar{w}_s, \tag{26}$$

with $D \equiv \left(\kappa^\xi \chi^{-\xi/k} \omega^{-1}\right)^\theta > 1$ comprising general equilibrium variables that are the same for all producers. It is easily checked in Eq. (26) that any equilibrium with $\hat{w}_n(\varphi_d) = w_n(\varphi_d)$, $\hat{w}_s(\varphi_o) =$

$w_s(\varphi_o)$, and firm-specific wage rates that are increasing in firm productivity must feature $u_n > 0$ and $u_s > 0$. Hence, in equilibrium firms pay the firm-specific fair wages both at home and abroad.

Whereas always strictly positive, the unemployment rate in the South decreases monotonically in $\hat{\eta}$, because Southern workers can only be employed in the production of offshored tasks. A richer pattern arises for the relationship between $\hat{\eta}$ and the unemployment rate in the North. From Eq. (26) it is immediate that the unemployment rate of Northern production workers can be written as

$$u_n = 1 - \frac{w_n(\varphi_d)}{\bar{w}_n}, \tag{27}$$

meaning it increases in wage dispersion, measured by the ratio of the average wage of employed workers and the wage paid by the marginal firm. The monotonic link between unemployment and wage dispersion is intuitive, because wage dispersion is the result of rent sharing, which in turn is the only source of labour market imperfection in our fair-wage model. Higher wage dispersion reflects a stronger labour market distortion from rent sharing, resulting in a higher rate of unemployed production workers. As we show in the Appendix, one can substitute for the endogenous wage ratio $w_n(\varphi_d)/\bar{w}_n$ to express $u_n$ as a function of model parameters and $\hat{\eta}$:

$$u_n = \frac{\theta\xi + (k-\xi)\left[1 - \delta(\hat{\eta})\right]}{\theta\xi + (k-\xi)}, \tag{28}$$

with $\delta(0) = 1$. For strictly positive but small values of $\hat{\eta}$ we have $\delta(\hat{\eta}) > 1$. The intuition for this result is straightforward: Starting from autarky, offshoring leads to the destruction of high-wage jobs in the North, and therefore to a reduction in the wage dispersion, which in turn reduces the labour market distortion from rent sharing and lowers the unemployment rate of production workers. We also show that $\delta(\hat{\eta}) < 1$ if $\hat{\eta}$ is high, because a further increase of $\hat{\eta}$ from an already high level leads to the creation of high-wage jobs in incumbent offshoring firms, increasing the wage dispersion and therefore exacerbating the labour market distortion from rent sharing. Hence, offshoring reduces the unemployment rate of domestic production workers relative to autarky if $\hat{\eta}$ is low, while the reverse is true $\hat{\eta}$ is high. We show in the Appendix that there exists a unique threshold $\hat{\eta}_u$, such that offshoring decreases (increases) the unemployment rate of production workers if $\hat{\eta} < (>) \hat{\eta}_u$.

Since only production workers can be unemployed in our model, the economy-wide unemployment rate in the North is given by $U_n \equiv u_n L/N_n$. We have shown above that the number of production workers falls monotonically in $\hat{\eta}$, due to changes in the occupational choice, and with $u_n$ decreasing in $\hat{\eta}$ as well if $\hat{\eta}$ is low, we know that unemployment $U_n$ definitely falls in this case. On the other hand, a further increase of $\hat{\eta}$ from an already high level is accompanied by an increase in $u_n$ and therefore the effect on economy-wide unemployment $U_n$ is not clear a priori in this case. We show in the Appendix that depending on parameter values indeed either effect can be dominant. The following proposition summarises our insights regarding the effects of offshoring on economy-wide unemployment.

**Proposition 2** *A reduction of offshoring costs lowers economy-wide unemployment in the North if $\hat{\eta}$ is low. An increase in unemployment relative to autarky is possible if $\hat{\eta}$ is high.*

**Proof** See the Appendix.

## 7 Conclusion

Empirical studies have found job polarisation to be pervasive across many developed economies over the past two decades. The leading explanation for this phenomenon is the routinisation hypothesis that builds on the idea that technological change and offshoring have made routine tasks obsolete and, provided that routine tasks are performed by workers with a medium skill level, this has caused a disappearance of jobs in the middle of the wage distribution. In this paper, we put forward an alternative explanation that is rooted in two well-documented empirical facts: the selection of more productive firms into offshoring; and the positive link between firm performance measures and wage payments. We propose an offshoring model that accords with these two observations and show that in our framework the decrease in variable offshoring costs implies that more firms choose offshoring, and they choose it for a wider range of tasks. At intermediate levels of offshoring costs this leads to job polarisation in the source country of offshoring, since the following three things happen: newly offshoring producers, which have intermediate productivity, reduce the number of domestic jobs, while incumbent offshoring producers with high productivity and low-productivity non-offshoring firms increase the number of domestic jobs under the then more advantageous labour market conditions. At least one of these three effects fails to materialise if offshoring costs are very high or very low, and hence our model allows us to describe the conditions under which job polarisation can be expected to occur.

Wage differences between firms in our model are the result of a labour market distortion from rent sharing that also leads to involuntary unemployment among production workers, and we show that changes in offshoring costs lead to non-monotonic effects on the rate of involuntary unemployment of production workers in the source country of offshoring. Accounting for the endogenous adjustment in the supply of production workers due to an occupational choice mechanism, we show that lower offshoring costs lead to lower economy-wide unemployment when offshoring costs are high, whereas the opposite can happen when offshoring costs are already low.

To the best of our knowledge, this paper provides the first attempt to highlight movements of workers between firms that offer different wages as an important channel through which offshoring can generate job polarisation. A first glance at firm-level data from Germany shows that the reallocation of workers across firms mirrors the creation and destruction of jobs along the wage distribution found by other studies. Whereas this evidence is only suggestive, we hope that our model can provide guidance for future empirical research to shed light on the importance of job polarisation between firms relative to the routinisation hypothesis that explains job polarisation by the movement of workers between jobs with differing task compositions.

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# A Appendix

## A.1 Derivation of Eq. (5)

In view of the Cobb-Douglas technology in Eq. (2) – which puts equal weights on all tasks – cost minimisation over $q(v,\eta)$ gives the well-known result that expenditures of a firm are the same for all of its tasks. This links the production of task $\eta<1$ to the production of task 1. We have

$$q(v,\eta)=\begin{cases}\left(\frac{1-\eta}{1-\hat{\eta}(v)}\right)^{-t}q(v,1) & \text{for all} \qquad \eta\leq\hat{\eta}(v)\\ q(v,1) & \text{for all} \qquad \eta>\hat{\eta}(v)\end{cases}, \tag{A.1}$$

where $\hat{\eta}(v)=0$ if firm $v$ performs all of its tasks domestically and $\hat{\eta}(v)>0$, otherwise. The total variable costs of producing intermediate goods output in quantity of $q(v)$ can then be expressed as $C(v)=q(v,1)w(v)$. According to Eq. (2), output of firm $v$ can be written as $q(v)=\varphi(v)q(v,1)\exp\{I(v)\ln\kappa[\hat{\eta}(v)]\}$, where $\kappa[\hat{\eta}(v)]$ is given by Eq. (6) and $I(v)$ is an indicator function with value 1 if the firm is engaged in offshoring and value 0, otherwise. The unit cost in Eq. (5) is finally obtained when dividing $C(v)$ by $q(v)$.

## A.2 Derivation of Eq. (15)

The global wage bill is the aggregate labour income of production workers in the North and the South and, in view of constant markup pricing, proportional to global operating profits:

$$(1-u_n)L\bar{w}_n+(1-u_s)N_s\bar{w}_s=N_n(\sigma-1)\left[\int_{\varphi_d}^{\varphi_o}\pi_d(\varphi)dG(\varphi)+\int_{\varphi_o}^{\infty}\pi_o(\varphi)dG(\varphi)\right]. \tag{A.2}$$

Making use of Eqs.(10) and (13) and accounting for $\pi_d(\varphi)/\pi_d(\varphi_d)=(\varphi/\varphi_d)^{\xi}$, we can compute

$$(1-u_n)L\bar{w}_n+(1-u_s)N_s\bar{w}_s=M(\sigma-1)\pi_d(\varphi_d)\frac{k}{k-\xi}(1+\chi). \tag{A.3}$$

In a similar vein, we can compute the wage bill accruing to domestic workers, according to

$$\begin{aligned}(1-u_n)L\bar{w}_n&=N_n(\sigma-1)\left[\int_{\varphi_d}^{\varphi_o}\pi_d(\varphi)dG(\varphi)+\hat{\eta}\int_{\varphi_o}^{\infty}\pi_o(\varphi)dG(\varphi)\right]\\&=M(\sigma-1)\pi_d(\varphi_d)\frac{k}{k-\xi}\left[1+(1-\hat{\eta})\chi-\hat{\eta}\chi^{\frac{k-\xi}{k}}\right].\end{aligned} \tag{A.4}$$

Dividing Eq. (A.4) by Eq. (A.3) and subtracting the resulting expression from 1 establishes $\gamma$ in Eq. (15). This completes the proof.

## A.3 Derivation of Eq. (17)

Combining Eqs. (15) and (A.4) gives

$$M(1+\chi)\pi_d(\varphi_d)\frac{k(\sigma-1)\gamma}{k-\xi}=(1-u_n)L\bar{w}_n. \tag{A.5}$$

Substituting $\pi_d(\varphi_d) = (1-u_n)\bar{w}_n$ from Eq. (11) and $(1+\chi)M = N_n - L$ from the resource constraint, we obtain

$$(N_n - L)\frac{k(\sigma-1)\gamma}{k-\xi} = L. \tag{A.6}$$

Solving Eq. (A.6) for $L$ establishes Eq. (17). This completes the proof.

### A.4 Derivation of Eqs. (20) and (21)

Combining Eqs. (1) and (2) with the assumption that one unit of labour input produces one unit of task output, we can infer

$$\frac{l_d(\varphi)}{l_d(\varphi_d)} = \frac{q_d(\varphi)/\varphi}{q_d(\varphi_d)/\varphi_d} = \left(\frac{p_d(\varphi)}{p_d(\varphi_d)}\right)^{-\sigma}\left(\frac{\varphi}{\varphi_d}\right)^{\sigma-1}. \tag{A.7}$$

Accounting for constant markup pricing and substituting Eq. (5) for $c(\varphi)$, and $w_d(\varphi)/w_d(\varphi_d) = (\varphi/\varphi_d)^{\theta\xi}$ from Eq. (8), we can compute

$$\ln l_d(\varphi) = \ln l_d(\varphi_d) + (1-\theta)\xi[\ln\varphi - \ln\varphi_d]. \tag{A.8}$$

Noting that constant markup pricing establishes $w_d(\varphi_d)l_d(\varphi_d) = (\sigma-1)\pi_d(\varphi_d)$, whereas the solution to the occupational choice problem in combination with the fair-wage constraint gives $w_d(\varphi_d) = \pi_d(\varphi_d)$, we have $l_d(\varphi_d) = \sigma - 1$. Furthermore, cutoff productivity $\varphi_d$ is linked to the mass of firms by means of $M/N_n = \varphi_d^{-k}$. To determine $M/N_n$, we can substitute Eq. (17) for $L$ in resource constraint $N_n = L + (1+\chi)M$. Solving the resulting expression for $M/N_n$ gives

$$\frac{M}{N_n} = \frac{k-\xi}{(1+\chi)[k-\xi+(1-\gamma)k(\sigma-1)]}. \tag{A.9}$$

Taking logs and substituting Eq. (15) for $\gamma$, we obtain $(1-\theta)\xi\ln\varphi_d = \mu(\hat{\eta})$, where $\mu(\hat{\eta})$ is given by Eq. (23). Substitution into Eq. (A.8) establishes Eq. (20). Thereby, we express firm-level employment (in logs) as a function of $\varphi$ and the market effects common to all firms, depending solely on the level of $\hat{\eta}$.

To determine the domestic employment level of offshoring firms, we make use of constant markup pricing to express domestic employment of domestic and offshoring producers as functions of their operating profits. This gives

$$l_d(\varphi) = (\sigma-1)\frac{\pi_d(\varphi)}{w_d(\varphi)} \qquad \text{and} \qquad l_o(\varphi) = (1-\hat{\eta})(\sigma-1)\frac{\pi_o(\varphi)}{w_o(\varphi)}, \tag{A.10}$$

respectively, using the fact that offshoring firms produce only a fraction $1-\hat{\eta}$ of their tasks domestically. Accounting for Eq. (10), we can express the firm-level employment effect of offshoring (in logs) as follows:

$$\ln l_o(\varphi) - \ln l_d(\varphi) = \ln(1-\hat{\eta}) + (1-\theta)\xi\ln\kappa. \tag{A.11}$$

Substituting Eq. (6′) for $\kappa$, we obtain $\lambda(\hat{\eta}) = \ln l_o(\varphi) - \ln l_d(\varphi)$ in Eq. (22). Eq. (21) is then obtained by adding $\lambda(\hat{\eta})$ to Eq. (20). This completes the proof.

### A.5 The properties of $\lambda(\hat{\eta})$ and a characterisation of $t_1$

Differentiating $\lambda(\hat{\eta})$, we can show that there exists a unique $\hat{\eta}_{tm} = 1/a$, with $a \equiv t(1-\theta)\xi$, such that $\lambda'(\hat{\eta}) >, =, < 0$ if $\hat{\eta} >, =, < \hat{\eta}_{tm}$. Then, $\hat{\eta}_{tm} < 1$ requires $a > 1$. However, this is not sufficient for $\hat{\eta}_{tm} < \hat{\eta}_{int}$. Combining Eqs. (6′) and (13), we can infer that $\hat{\eta}_{tm} = \hat{\eta}_{int}$ is reached if

$$\Gamma(a) = -\frac{1}{1-\theta}\left[a \ln\left(\frac{a-1}{a}\right) + 1\right] - \ln 2 = 0, \tag{A.12}$$

where $\Gamma(a) = 0$ can be transformed into Eq. (24), when substituting $a = t(1-\theta)\xi$. Since $\lim_{a\to 1}\Gamma(a) = \infty$, $\lim_{a\to\infty}\Gamma(a) = -\ln 2$ and $\Gamma'(a) < 0$, we know that $\Gamma(a) = 0$ has a unique solution in $a$ and, in extension, also in $t$, which we denote by $t_1$. Finally, $\hat{\eta}_{tm} < \hat{\eta}_{int}$ requires $\Gamma(a) < 0$ and thus $t > t_1$. This completes the proof.

### A.6 Properties of $\mu(\hat{\eta})$ and characterisation of $\hat{\eta}_p^1$

From Eqs. (6′) and (13), we can infer that

$$\chi(\hat{\eta}) = \left\{(1-\hat{\eta})^{-t\xi}\exp[-t\xi\hat{\eta}] - 1\right\}^{\frac{k}{\xi}}, \qquad \chi'(\hat{\eta}) = tk\chi(\hat{\eta})^{\frac{k-\xi}{k}}(1+\chi^{\xi/k})\frac{\hat{\eta}}{1-\hat{\eta}}. \tag{A.13}$$

Differentiation of $\mu(\hat{\eta})$ then gives

$$\mu'(\hat{\eta}) = -\frac{(1-\theta)\xi}{k(k-\xi)}\frac{\chi'(\hat{\eta})f(\hat{\eta})}{1+\chi(\hat{\eta}) + [k(\sigma-1)/(k-\xi)]\left[1+(1-\hat{\eta})\chi(\hat{\eta}) - \hat{\eta}\chi(\hat{\eta})^{\frac{k-\xi}{k}}\right]}, \tag{A.14}$$

with

$$f(\hat{\eta}) \equiv k - \xi + k(\sigma-1)\left[1-\hat{\eta} - \frac{1}{tk}\frac{1-\hat{\eta}}{\hat{\eta}} - \frac{k-\xi}{k}\hat{\eta}\chi(\hat{\eta})^{-\frac{\xi}{k}}\right]. \tag{A.15}$$

In view of $\chi'(\hat{\eta}) > 0$, it is immediate that $\mu'(\hat{\eta}) >, =, < 0$ if $0 >, =, < f(\hat{\eta})$. From $\lim_{\hat{\eta}\to 0} f(\hat{\eta}) = -\infty$, we can thus conclude that $\mu(\hat{\eta})$ increases at low levels of $\hat{\eta}$. This leaves two possible outcomes: $\mu'(\hat{\eta}) > 0$ for all possible $\hat{\eta}$; $\mu'(\hat{\eta}) = 0$ at some $\hat{\eta} \in (0,1)$. In the latter case, $\mu(\hat{\eta})$ has an interior extremum, and we show in the following that if an interior extremum exists, it is unique and, in view of $\mu'(0) > 0$, a maximum. This is equivalent to showing that $f(\hat{\eta}) = 0$ has a unique interior solution, provided that it has a solution at all.

Differentiation of $f(\hat{\eta})$ gives

$$f'(\hat{\eta}) = -\frac{k(\sigma-1)}{1-\hat{\eta}}\left\{1-\hat{\eta} - \frac{1}{tk}\frac{1-\hat{\eta}}{\hat{\eta}} - \frac{k-\xi}{k}\hat{\eta}\chi(\hat{\eta})^{-\frac{\xi}{k}} - \frac{1}{tk}\left(\frac{1-\hat{\eta}}{\hat{\eta}}\right)^2 + \frac{k-\xi}{k}\chi^{-\frac{\xi}{k}}g(\hat{\eta})\right\}, \tag{A.16}$$

with $g(\hat{\eta}) \equiv 1 - t\xi\hat{\eta}^2\chi(\hat{\eta})^{-\xi/k}[1+\chi(\hat{\eta})^{\xi/k}]$. Evaluated at $f(\hat{\eta}) = 0$, we find that the first three terms in the bracket expression add up to $[f(\hat{\eta}) - (k-\xi)]/[k(\sigma-1)] = -(k-\xi)/[k(\sigma-1)] < 0$. Thus, $g(\hat{\eta}) \le 0$ is sufficient for an extremum of $\mu(\hat{\eta})$ to be a maximum and, if $g(\hat{\eta}) \le 0$ holds for all possible $\hat{\eta}$, this maximum must be unique. Substituting $\chi(\hat{\eta})$ from the main text we can compute $\lim_{\hat{\eta}\to 0} g(\hat{\eta}) = -1$ and $g(1) = 1 - t\xi < 0$, where the negative sign of $g(1)$ follows from assumption $t > t_1$ and the observation that $t > t_1$ requires $t(1-\theta)\xi > 1$ (see Appendix A.5). Furthermore, we can compute $g'(\hat{\eta}) = \left\{2 + [g(\hat{\eta}) - 1](1-\hat{\eta})^{t\xi-1}\exp[\hat{\eta}t\xi]\right\}[g(\hat{\eta}) - 1]/\hat{\eta}$. Since $g(\hat{\eta}) = 1$ is ruled out for any $\hat{\eta} \geq 0$, we can readily conclude that $g'(\hat{\eta}) = 0$ is only possible if $g(\hat{\eta}) < 0$. Accordingly, $g(\hat{\eta})$ is either monotonic in $\hat{\eta}$ or has a (not necessarily unique) extremum with negative function value. In both cases, $g(0) = -1$ and $g(1) = 1 - t\xi < 0$ are sufficient for $g(\hat{\eta})$ to have a negative sign for all possible $\hat{\eta}$. Together with our earlier results this implies that $\mu(\hat{\eta})$ has unique maximum, which is either in the interior of interval $(1, \hat{\eta}_{int})$ and determined by $\mu'(\hat{\eta}) = 0$ or it is given by the corner solution $\hat{\eta}_{int}$ if $\mu'(\hat{\eta}) > 0$ for all possible $\hat{\eta}$. In both cases, we denote the maximum of $\mu(\hat{\eta})$ by $\hat{\eta}_p^1$. This completes the proof.

## A.7 Characterisation of $t_2$ and the ranking of $\hat{\eta}_p^1$ and $\hat{\eta}_{tm}$

We now show that under parameter constraint (25), there exists a unique $t_2$, such that $0 >, =, < f(\hat{\eta}_{tm})$ and thus $\mu'(\hat{\eta}_{tm}) >, =, < 0$ if $t >, =, < t_2$. Evaluating $f(\hat{\eta})$ at $\hat{\eta}_{tm} = [t(1-\theta)\xi]^{-1}$ gives $f(\hat{\eta}_{tm}) = (k-\xi)[1 + (\sigma-1)f_0(a)]$, with

$$f_0(a) \equiv \left[\frac{k-(1-\theta)\xi}{k-\xi} - \frac{1}{n(a)}\right]\frac{a-1}{a}, \quad n(a) \equiv (a-1)\left\{\left(\frac{a-1}{a}\right)^{-\frac{a}{1-\theta}} \exp\left[-\frac{1}{1-\theta}\right] - 1\right\},$$

and $a \equiv t(1-\theta)\xi > 1$. Hence, $f(\hat{\eta}_{tm}) < 0$ requires $1 + (\sigma-1)f_0 < 0$. Function $f_0(a)$ has the following two properties, which we formally show in a technical supplement that is available upon request:

1. There exists a unique $\underline{a} \in (1, \infty)$, such that $f_0(a) >, =, < 0$ if $\underline{a} >, =, < a$;

2. If $f_0(a)$ has an extremum at $a > \underline{a}$, this must be a minimum.

From property 1 it follows that $\lim_{a\to\infty} f_0(a) < 0$, which is equivalent to $1 - \theta + \theta k/(k-\xi) > 0$. Accordingly, the formal condition for $1 + (\sigma-1)\lim_{a\to\infty} f_0(a) < 0$ is given by (25). Furthermore, from property 2, we can infer that either $f_0(a)$ is negatively sloped over the whole interval $(\underline{a}, \infty)$ or there exists a unique $\hat{a}_0 > \underline{a}$, such that $f_0(a)$ decreases over subinterval $(\underline{a}, \hat{a}_0)$ and increases over subinterval $(\hat{a}_0, \infty)$. Under parameter constraint (25), condition $1 + (\sigma-1)f_0(a_2) = 0$ therefore determines a unique $t_2 = a_2/[(1-\theta)\xi]$, such that $0 >, =, < 1 + (\sigma-1)f_0(a)$ and thus $\mu(\hat{\eta}_{tm}) >, =, < 0$ if $t >, =, < t_2$. This completes the proof.

## A.8 Characterisation of $\hat{\eta}_p^0$

Let $\hat{\eta}_p^0$ be implicitly defined by $\lambda'(\hat{\eta}) + \mu'(\hat{\eta}) = 0$. Then, existence of $\hat{\eta}_p^0 \in (0, \hat{\eta}_{tm})$ is guaranteed if (i) $\lambda'(0) + \mu'(0) < 0$ and (ii) $\lambda'(\hat{\eta}_{tm}) + \mu'(\hat{\eta}_{tm}) > 0$ hold. The second condition follows from the observation that (by definition) $\lambda'(\hat{\eta}_{tm}) = 0$ and $\mu'(\hat{\eta}_{tm}) > 0$ if parameter constraint (25) and $t > \max\{t_1, t_2\}$ hold. To show that the first condition is also fulfilled, we can compute

$$\begin{aligned}\lambda'(\hat{\eta}) + \mu'(\hat{\eta}) = &-[1 - t(1-\theta)\xi]\frac{1}{1-\hat{\eta}} - t(1-\theta)\xi \\ &- \frac{(1-\theta)\xi}{k}\frac{\chi'(\hat{\eta})f(\hat{\eta})}{(k-\xi)[1+\chi(\hat{\eta})] + k(\sigma-1)\left[1 + (1-\hat{\eta})\chi(\hat{\eta}) - \hat{\eta}\chi(\hat{\eta})^{\frac{k-\xi}{k}}\right]},\end{aligned} \tag{A.17}$$

according to Eqs. (22) and (A.14). This establishes

$$\lim_{\hat{\eta}\to 0}\left[\lambda'(\hat{\eta})+\mu'(\hat{\eta})\right]=-1-\frac{(1-\theta)\xi}{k(k\sigma-\xi)}\lim_{\hat{\eta}\to 0}\chi'(\hat{\eta})f(\hat{\eta}). \tag{A.18}$$

From Eq. (A.13) it follows that $\lim_{\hat{\eta}\to 0}\chi'(\hat{\eta})=0$. Substituting Eq. (A.15) for $f(\hat{\eta})$, we can then compute

$$\begin{aligned}\lim_{\hat{\eta}\to 0}\left[\lambda'(\hat{\eta})+\mu'(\hat{\eta})\right]&=-1-\frac{t(1-\theta)\xi(\sigma-1)(k-\xi)}{k\sigma-\xi}\lim_{\hat{\eta}\to 0}\frac{\hat{\eta}^2}{1-\hat{\eta}}\chi(\hat{\eta})^{1-2\frac{\xi}{k}},\\&=-1-\frac{2(1-\theta)(\sigma-1)(k-\xi)}{k\sigma-\xi}\lim_{\hat{\eta}\to 0}\left\{(1-\hat{\eta})^{-t\xi}\exp[-\hat{\eta}t\xi]-1\right\}^{\frac{k-\xi}{\xi}},\end{aligned} \tag{A.19}$$

which establishes $\lambda'(0)+\mu'(0)=-1$ and completes the proof.

## A.9 Derivation of Eq. (28)

Adding up domestic employment over all purely domestic and offshoring firms in the source country gives

$$(1-u_n)\,L=N_n\left[\int_{\varphi_d}^{\varphi_o}l_d(\varphi)\,dG(\varphi)+\int_{\varphi_o}^{\infty}l_o(\varphi)\,dG(\varphi)\right]. \tag{A.20}$$

Substituting Eqs. (20) and (21) and using $l_d(\varphi_d)=\sigma-1$, we can compute

$$(1-u_n)L=Ml_d(\varphi_d)\beta(\hat{\eta})\frac{k}{k-(1-\theta)\,\xi}, \tag{A.21}$$

with $\beta(\hat{\eta})\equiv 1+\chi(\hat{\eta})^{1-(1-\theta)\xi/k}\left\{\exp[\lambda(\hat{\eta})]-1\right\}$. Furthermore, making use of Eq. (A.4) and noting that constant markup pricing implies $(\sigma-1)\pi(\varphi_d)=l_d(\varphi_d)w_n(\varphi_d)$, we can express the total wage bill in the North as follows:

$$(1-u_n)L\bar{w}_n=Ml_d(\varphi_d)w_n(\varphi_d)(1-\gamma)[1+\chi(\hat{\eta})]\frac{k}{k-\xi}, \tag{A.22}$$

Together Eqs. (A.21) and (A.22) determine the wage ratio $w(\varphi_d)/\bar{w}=\delta(\hat{\eta})(k-\xi)/[k-(1-\theta)\xi]$, where we have used $\delta(\hat{\eta})\equiv\beta(\hat{\eta})/\left\{(1-\gamma)[1+\chi(\hat{\eta})]\right\}$. Substitution into Eq. (27) then establishes Eq. (28).

## A.10 Offshoring and the unemployment rate of production workers $u_n$

From Eq. (28), we can infer that offshoring decreases (increases) unemployment rate $u_n$ relative to autarky, if $\delta(\hat{\eta})>(<)1$. Substituting Eqs. (15), (22), and (A.13) for $\gamma$, $\lambda(\hat{\eta})$, and $\chi(\hat{\eta})$, respectively, and accounting for $\beta(\hat{\eta})$ from Appendix A.9, it follows that $\delta(\hat{\eta})>,=,<1$ is equivalent to

$$\begin{aligned}&\left\{(1-\hat{\eta})^{-t\xi}\exp[-t\xi\hat{\eta}]-1\right\}^{\frac{k-\xi}{\xi}+\theta}\left\{(1-\hat{\eta})^{1-t(1-\theta)\xi}\exp[-\hat{\eta}t(1-\theta)\xi]-1\right\}\\&\qquad>,=,<\left\{(1-\hat{\eta})^{-t\xi}\exp[-\hat{\eta}t\xi]-1\right\}^{\frac{k-\xi}{\xi}}\left\{(1-\eta)^{1-t\xi}\exp[-\hat{\eta}t\xi]-1\right\}.\end{aligned} \tag{A.23}$$

Let us define

$$\psi(\hat{\eta}) \equiv \left\{(1-\hat{\eta})^{-t\xi}\exp[-\hat{\eta}t\xi]-1\right\}^{\theta}, \qquad \psi_1(\hat{\eta}) \equiv (1-\hat{\eta})^{1-t\xi}\exp[-\hat{\eta}t\xi]-1, \tag{A.24}$$

$$\psi_2(\hat{\eta}) \equiv (1-\hat{\eta})^{1-t(1-\theta)\xi}\exp[-\hat{\eta}t(1-\theta)\xi]-1. \tag{A.25}$$

Then, $\psi_1(\hat{\eta}) = 0$ characterises a unique $\hat{\eta}_b^1 > 0$, such that $\psi_1(\hat{\eta}) >,=,< 0$ if $\hat{\eta} >,=,< \hat{\eta}_b^1$. Furthermore, we know from the main text that $\psi_2(\hat{\eta})$ has a unique minimum at $\hat{\eta}_{tm}$, whereas $\psi_2(\hat{\eta}) = 0$ characterises a unique $\hat{\eta}_b^2 > \hat{\eta}_b^1$, such that $\psi_2(\hat{\eta}) >,=,< 0$ if $\hat{\eta} >,=,< \hat{\eta}_b^2$. It is worth noting that $t > \max\{t_1, t_2\}$ is sufficient for $\hat{\eta}_{tm} < \hat{\eta}_{int}$, but not for $\hat{\eta}_b^1, \hat{\eta}_b^2 < \hat{\eta}_{int}$.

Let us now define $\hat{\psi}(\hat{\eta}) \equiv \psi_1(\hat{\eta})/\psi_2(\hat{\eta})$. We can then infer from (A.23) that $\delta(\hat{\eta}) >,=,< 1$ is equivalent to $\hat{\psi}(\hat{\eta}) >,=,< \psi(\hat{\eta})$ if $\hat{\eta} < \hat{\eta}_b^2$, whereas $\delta(\hat{\eta}) >,=,< 1$ is equivalent to $\psi(\hat{\eta}) >,=,< \hat{\psi}(\hat{\eta})$ if $\hat{\eta} > \hat{\eta}_b^2$. We can distinguish the following cases, regarding the sign and size of $\hat{\psi}(\hat{\eta})$: (i) $0 > \psi_1(\hat{\eta}) > \psi_2(\hat{\eta})$ and thus $\hat{\psi}(\hat{\eta}) \in (0,1)$ if $\hat{\eta} < \hat{\eta}_b^1$; (ii) $\psi_1(\hat{\eta}) > 0 > \psi_2(\hat{\eta})$ and thus $\hat{\psi}(\hat{\eta}) < 0$ if $\hat{\eta} \in (\hat{\eta}_b^1, \hat{\eta}_b^2)$; (iii) $\psi_1(\hat{\eta}) > \psi_2(\hat{\eta}) > 0$ and thus $\hat{\psi}(\hat{\eta}) > 1$ if $\hat{\eta} > \hat{\eta}_b^2$.[16] We can thus safely conclude that $\psi(\hat{\eta}) > \hat{\psi}(\hat{\eta})$ and thus $\delta(\hat{\eta}) < 1$ if $\hat{\eta} \in (\hat{\eta}_b^1, \hat{\eta}_b^2)$, whereas $\psi(\hat{\eta}) < \hat{\psi}(\hat{\eta})$ and thus $\delta(\hat{\eta}) < 1$ if $\hat{\eta} > \hat{\eta}_b^2$ and at the same time $\hat{\eta} < \hat{\eta}_{int}$. The latter follows from the observation that $\psi(\hat{\eta}) < 1$ for all $\hat{\eta} < \hat{\eta}_{int}$. In both cases (and hence whenever $\hat{\eta} \geq \hat{\eta}_b^1$), the unemployment rate of production workers in the North is higher under offshoring than in autarky. We now look at the remaining domain and note first that $\psi(\hat{\eta})$ increases over interval $(0, \hat{\eta}_{int})$ from a minimum level of $\psi(0) = 0$ to a maximum level of $\psi(\hat{\eta}_{int}) = 1$. Hence, showing that $\hat{\psi}(\hat{\eta})$ falls monotonically over interval $[0, \hat{\eta}_b^1]$ from a maximum level of $\hat{\psi}(0) = 1$ to a minimum level of $\hat{\psi}\left(\hat{\eta}_b^1\right) = 0$, suffices to prove that $\psi(\hat{\eta}) = \hat{\psi}(\hat{\eta})$ has a unique solution $\hat{\eta}_u < \hat{\eta}_{int}$, such that $\psi(\hat{\eta}) >,=,< \hat{\psi}(\hat{\eta})$ if $\hat{\eta} >,=,< \hat{\eta}_u$. This can be done by using higher differentials of $\hat{\psi}(\hat{\eta})$ and a detailed proof for this result is provided in a technical supplement, which is available upon request. We can thus safely conclude that $1 >,=,< \delta(\hat{\eta})$ if $\hat{\eta} >,=,< \hat{\eta}_u$. This completes the proof.

## A.11 Proof of Proposition 2

We can write the economy-wide unemployment in the North as $U_n = \Lambda U_n^a$, with

$$\Lambda(\hat{\eta}) \equiv \frac{u_n L}{u_n^a L^a} = \frac{\theta\xi + (k-\xi)\left[1-\delta(\hat{\eta})\right]}{\theta\xi}\frac{[k-\xi+k(\sigma-1)](1-\gamma)}{k-\xi+k(\sigma-1)(1-\gamma)} \tag{A.26}$$

and superscript $a$ referring to autarky. That offshoring increases the economy-wide rate of unemployment in the North at low levels of $\hat{\eta}$ then follows from the observation that $\delta(\hat{\eta}) > 1$ and thus $u < u^a$ if $\hat{\eta} < \hat{\eta}_u$ (see above) and the fact that the number of production workers decreases monotonically in $\hat{\eta}$. Furthermore, the economy-wide rate of unemployment $U$ is larger (smaller) at $\hat{\eta} = \hat{\eta}_{int}$ and thus $\chi = 1$ than under autarky if $\Lambda(\hat{\eta}_{int}) > (<) 1$, which is equivalent to

$$1 - \frac{\theta\xi}{\theta\xi + (1-2^{-\theta})(k\sigma - \xi)} > (<)\hat{\eta}_{int}. \tag{A.27}$$

Taking into account that $\hat{\eta}_{int}$ is implicitly defined as function of $t$ by $\chi = 1$ and thus by $\{(1-\hat{\eta}_{int})^{-t\xi}\exp[-\hat{\eta}_{int}t\xi]-1\}^{k/\xi} = 1$, it follows that $\lim_{t\to 0}\hat{\eta}_{int} = 1$, $\lim_{t\to\infty}\hat{\eta}_{int} = 0$, and $d\hat{\eta}_{int}/dt < 0$.

[16] For completeness, we also have $\lim_{\hat{\eta}\to 0}\hat{\psi}(\hat{\eta}) = 1$, $\hat{\psi}(\hat{\eta}_b^1) = 0$, $\lim_{\hat{\eta}\to\hat{\eta}_b^{2-}}\hat{\psi}(\hat{\eta}) = -\infty$, and $\lim_{\hat{\eta}\to\hat{\eta}_b^{2+}}\hat{\psi}(\hat{\eta}) = \infty$.

Hence, there exists a unique $t$ for which (A.27) holds with equality. This critical $t$ is denoted $t_U$ and given by

$$t_U \equiv \frac{\ln 2}{\xi\,(1-\zeta-\ln\zeta)}, \qquad \zeta \equiv \frac{\theta\xi}{\theta\xi+(1-2^{-\theta})[k\sigma-\xi]}. \tag{A.28}$$

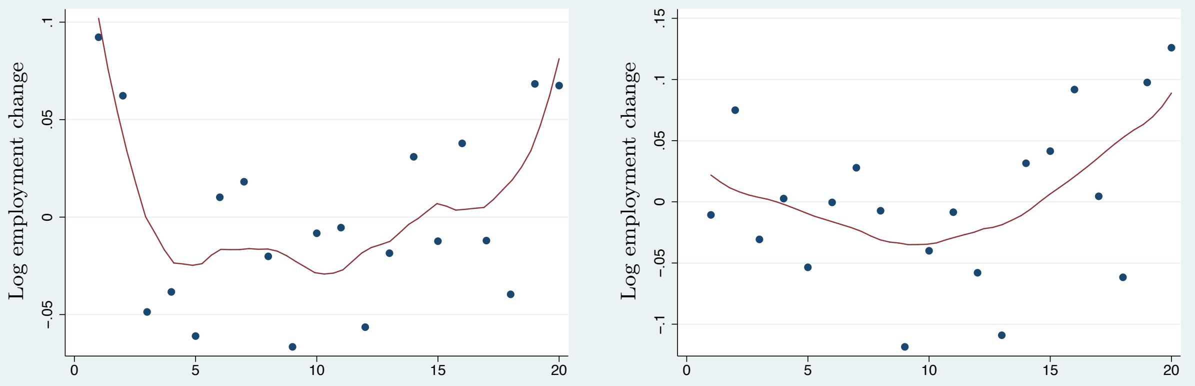
For $t >, =, < t_U$, we have $\Lambda(\hat{\eta}_{int}) >, =, < 1$. This completes the proof.

# S Supplement (for online publication)

## S.1 Further descriptives: Robustness of Figure 1

In this supplement, we present further descriptive evidence for a u-shape in the employment changes of German establishments over the wage distribution. For this purpose, we use establishment information from the Linked Employer-Employee Dataset of the Institute for Employment Research in Nuremberg (LIAB) and consider all establishments from this dataset, for which we have the relevant information in the base and the end year of the observation period.[17] We proceed as in the main text, rank firms by their mean wages in 1999, assign them to groups of symmetric size, and display employment changes for each establishment group (in log difference), which we normalise by the overall trend in the data in order to control for macroeconomic shocks that are common to all establishments. To highlight the u-shape, we run a regression based on local polynominal smoothing.[18] We rely on a smoothing algorithm that is based on Epanechnikov kernel-weighting with degree 1 and determine the optimal bandwidth by using the rule-of thumb method of bandwidth selection that minimizes the conditional weighted mean integrated squared error (see Fan and Gijbels, 1996).

In the first two robustness checks depicted by Figure S.1, we change the length of the observation period. In Panel A, we consider the shorter period of 1999-2003. Whereas this increases the number of observations in the balanced establishment panel to 7,584, reducing the time span may make the data more vulnerable to business cycle effects, in particular if these effects are different for establishments from different wage categories. In Panel B, we consider the longer period of 1999-2007. This reduces the number of establishments to 4,442. Both exercises provide further support for a job polarisation across firms as displayed in Figure 1.




*Panel A: Observation period 1999-2003* *Panel B: Observation period 1999-2007*

*Source*: LIAB dataset from the Institute for Employment Research in Nuremberg. Dots capture employment changes for 20 establishment groups (of equal size); Solid lines depict the outcome of local polynominal smoothing, using Epanechnikov kernel-weighting with degree 1 and (rule-of-thumb) bandwidth 1.37 for Panel A and 2.59 for Panel B.

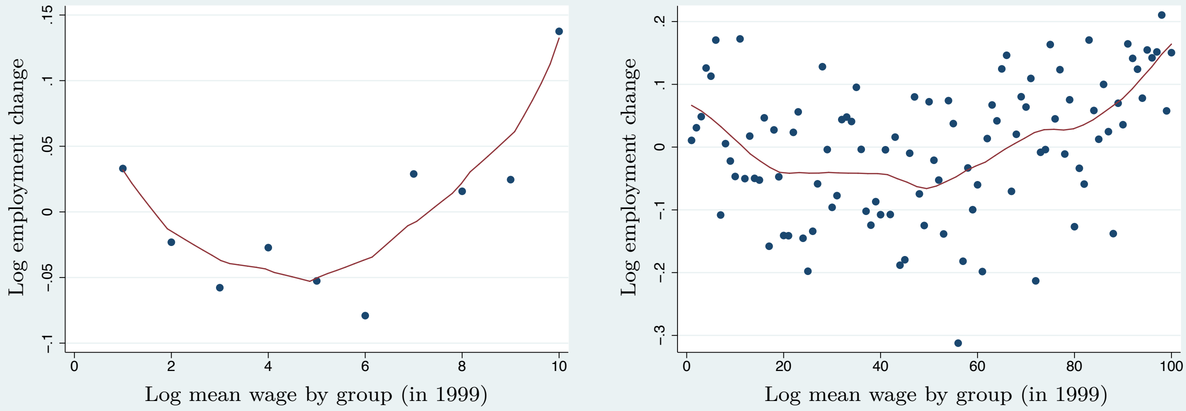
**Figure S.1:** *Job polarisation between German establishments – Alternative time spans*

In a further set of robustness checks displayed in Figure S.2, we keep the initial observation period of 1999-2005 but change the number of wage groups to which we assign the 5,754 establishments from the data. In Panel A, we assign the establishments to 10 groups (of equal size) and thus

[17]For detailed information on how the LIAB data is constructed, see Klein et al. (2013).

[18]A clear advantage of local polynomial smoothing is that it performs better than other smoothing methods near the bounds of the data interval.

build the analysis of employment changes on more aggregated data. In Panel B, we consider 100 different establishment groups and thus rely on more disaggregated data. Both exercises provide further supportive evidence for a job polarisation across German establishments over the period 1999-2005.

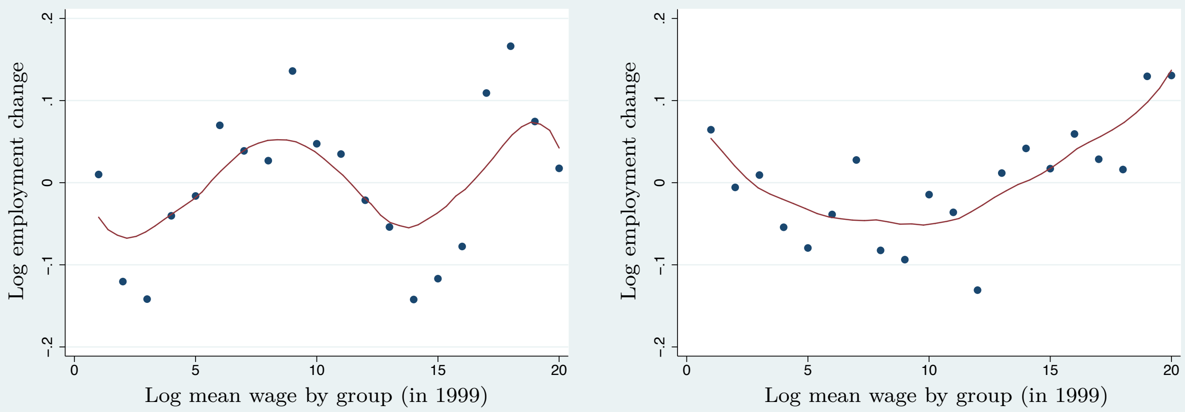



*Panel A: 10 establishment groups* *Panel B: 100 establishment groups*

*Source*: LIAB dataset from the Institute for Employment Research in Nuremberg. Dots capture employment changes for 10/100 establishment groups (of equal size); Solid lines depict the outcome of local polynominal smoothing, using Epanechnikov kernel-weighting with degree 1 and (rule-of-thumb) bandwidth 0.93 for Panel A and 6.53 for Panel B.

**Figure S.2:** *Job polarisation between German establishments – Alternative group specifications*

Figure S.3 summarises the insights from two additional experiments. In Panel A, we use establishment information on offshoring for the year 1999 to restrict the analysis to incumbent offshorers, i.e. to those 1,576 establishments in our dataset that conducted offshoring already in the first observation period, and we report for the period 1999-2005 the employment changes for this subsample of establishments. We see that a u-shape in the employment changes along the wage distribution is not observed for incumbent offhorers and conclude from this observation that asymmetric behavior of offshoring and non-offshoring firms is crucial for the existence of job polarisation in Germany. In Panel B, we use the same data as in Figure 1 but exclude entrepreneurs and managers from the worker sample. This leaves the pattern of job polarisation unaffected.




*Panel A: Incumbent offshorers only* *Panel B: Exclusion of entrepreneurs and managers*

*Source*: LIAB dataset from the Institute for Employment Research in Nuremberg. Dots capture employment changes for 20 establishment groups (of equal size); Solid lines depict the outcome of local polynominal smoothing. Solid lines depict the outcome of local polynominal smoothing, using Epanechnikov kernel-weighting with degree 1 and (rule-of-thumb) bandwidth 1.86 for Panel A and 1.87 for Panel B.

**Figure S.3:** *Job polarisation between German establishments – Further robustness checks*

## S.2 Background material for Appendix A.7: The properties of $f_0(a)$

Let us first note that $f_0(1) = 0$ and $\lim_{a\to\infty} f_0(a) = -[1-\theta-\theta k/(k-\xi)]$, which is negative under parameter constraint (25). Twice differentiating $f_0(a)$ furthermore gives

$$f_0'(a) = \left[\frac{k-(1-\theta)\xi}{k-\xi} - \frac{1}{n(a)}\right]\left(\frac{1}{a}\right)^2 + \frac{n'(a)}{n(a)^2}\frac{a-1}{a}, \tag{S.1}$$

$$f_0''(a) = -2\left[\frac{k-(1-\theta)\xi}{k-\xi} - \frac{1}{n(a)}\right]\left(\frac{1}{a}\right)^3 + 2\frac{n'(a)}{n(a)^2}\left(\frac{1}{a}\right)^2 - 2\frac{[n'(a)]^2}{n(a)^3}\frac{a-1}{a} + \frac{n''(a)}{n(a)^2}\frac{a-1}{a}, \tag{S.2}$$

with $\lim_{a\to1} f_0'(a) = [k-(1-\theta)\xi]/(k-\xi) > 0$ and $\lim_{a\to\infty} f_0'(a) = 0$. Combining $f_0(1) = 0$ and $f_0'(1) > 0$ with $\lim_{a\to\infty} f_0(a) < 0$ has the following two implications. On the one hand, there must be an $\underline{a} \in (0,1)$ for which $f_0(\underline{a}) = 0$ and $f_0'(\underline{a}) < 0$ and, on the other hand, $f_0(a)$ must have a maximum on interval $(0, \underline{a})$. In the following, we show that $f_0(a)$ has a unique maximum on interval $(0,\infty)$, implying that $\underline{a}$ must be unique as well, and that any extremum in interval $(\underline{a},\infty)$ must be a minimum.

For this purpose, we note that an extremum of $f_0(a)$ requires $f_0'(a) = 0$ and denote by $\hat{a}_0$ those values of $a$ that fulfill this condition. We then compute $f_0''(\hat{a}_0) = f_1(\hat{a}_0)[n(\hat{a}_0)]^{-3}\hat{a}_0^{-1}$, with

$$f_1(a) = 2n'(a)n(a) - (a-1)[n'(a)]^2 + (a-1)n''(a)n(a). \tag{S.3}$$

Considering $n(a)$ from Appendix A.7, we can compute

$$n'(a) = -\left(\frac{a-1}{a}\right)^{-\frac{a}{1-\theta}} \exp\left[-\frac{1}{1-\theta}\right]\left[(a-1)\ln\left(\frac{a-1}{a}\right)+\theta\right]\frac{1}{1-\theta} - 1, \tag{S.4}$$

$$n''(a) = \left(\frac{a-1}{a}\right)^{-\frac{a}{1-\theta}} \exp\left[-\frac{1}{1-\theta}\right]\frac{1}{1-\theta}\hat{n}(a), \tag{S.5}$$

with

$$\hat{n}(a) \equiv \left[\ln\left(\frac{a-1}{a}\right) + \frac{1}{a-1}\right]^2\frac{a-1}{1-\theta} - 2\ln\left(\frac{a-1}{a}\right) - \frac{2a-1}{a(a-1)}. \tag{S.6}$$

This establishes $f_1(a) = f_2(a)[n(a)+(a-1)]/[a(1-\theta)]$, with

$$f_2(a) \equiv -\left[\left(\frac{a-1}{a}\right)^{-\frac{a}{1-\theta}}\exp\left[-\frac{1}{1-\theta}\right]+1\right]\left\{\left[(a-1)\ln\left(\frac{a-1}{a}\right)+1\right]^2\frac{a}{1-\theta}-1\right\} - 2. \tag{S.7}$$

For the subsequent analysis, it is useful to introduce

$$v_1(a) \equiv (a-1)\ln\left(\frac{a-1}{a}\right) + 1 \ge 0, \qquad v_2(a) \equiv 3\ln\left(\frac{a-1}{a}\right) + \frac{3a-1}{a(a-1)} \ge 0, \tag{S.8}$$

$$v_3(a) \equiv (3a-1)\ln\left(\frac{a-1}{a}\right) + 3 \le 0, \qquad v_4(a) \equiv (4a-1)\ln\left(\frac{a-1}{a}\right) + 4 \le 0, \tag{S.9}$$

with $v_2(a) = v_3'(a)$. We can compute $\lim_{a\to1} f_2(a) = -\infty$, $\lim_{a\to\infty} f_2(a) = 0$, and

$$f_2'(a) = \frac{1}{1-\theta}\frac{v_1(a)}{a-1}\left\{ \left(\frac{a-1}{a}\right)^{-\frac{a}{1-\theta}} \exp\left[-\frac{1}{1-\theta}\right]\left[v_1(a)^2\frac{a}{1-\theta} - 1\right] - \left[\left(\frac{a-1}{a}\right)^{-\frac{a}{1-\theta}} \exp\left[-\frac{1}{1-\theta}\right] + 1\right](a-1)v_3(a)\right\}.$$

Let us now hypothesise that $f_2(a) = 0$ has a solution $a_0(\theta) > 1$ ($a_0$, in short). Then, $f_2'(a_0) >, =, < 0$ is equivalent to $F(a_0, \theta) >, =, < 0$, with

$$F(a, \theta) \equiv \left[v_1(a)^2\frac{a}{1-\theta}\right]^2 - 1 - 2(a-1)v_3(a). \tag{S.10}$$

Differentiation of $F(a)$ establishes

$$F_a'(a,\theta) = 2\left\{\left[v_1(a)^3\frac{a}{(1-\theta)^2} - 1\right]v_3(a) - (a-1)v_2(a)\right\}, \tag{S.11}$$

$$F_{aa}''(a,\theta) = 2\left\{\left[v_1(a)^3\frac{a}{(1-\theta)^2} - 2\right]v_2(a) + \frac{1}{(1-\theta)^2}v_1(a)^2v_3(a)v_4(a) + \frac{a+1}{a^2(a-1)}\right\}. \tag{S.12}$$

If $F_a'(a,\theta) = 0$ has a solution at $\underline{a}_1(\theta) > 1$ ($\underline{a}_1$, in short). Then, $F''(\underline{a}_1, \theta) >, =, < 0$ is equivalent to $F_0(\underline{a}_1) >, =, < 0$, with

$$F_0(a) \equiv -\frac{2}{av_1(a)v_3(a)}\left\{\left[\left(2a\ln\left(\frac{a-1}{a}\right) + 1\right)v_1(a) - (a-1)v_3(a)v_4(a)\right]v_2(a) - \left[v_3(a)v_4(a) + \frac{a+1}{a(a-1)}v_1(a)\right]v_3(a)\right\} \tag{S.13}$$

being independent of $\theta$. It is tedious but straightforward to show that $\lim_{a\to1} F_0(a) = -\infty$, $\lim_{a\to\infty} F_0(a) = 0$, and that $F_0(a) = 0$ has a unique solution at $\underline{a}_2 > 1$, such that $F_0(a) >, =, < 0$ if $a >, =, < \underline{a}_2$. The correctness of this result can be verified with any standard mathematics software.

We can further note that $\lim_{a\to1} F_a'(a,\theta) = -\infty$, and $\lim_{a\to\infty} F_a'(a,\theta) = 0$ hold for all $\theta \in (0,1)$. Let us assume that $F_a'(a,\theta) = 0$ has a solution at $\underline{a}_1(\theta) \in [1,\infty)$. For instance, this is the case if $\theta \to 0$. Accounting for $\lim_{a\to1} F_a'(a,\theta) = -\infty$ and $F_0(a) >, =, < 0$ if $a >, =, < \underline{a}_2$, it follows that $\underline{a}_1(\theta)$ is unique and larger than $\underline{a}_2$, i.e. $\underline{a}_1(\theta) > \underline{a}_2 > 1$. However, this insight builds on the presumption that $F_a'(a,\theta) = 0$ has in fact a solution on interval $[1,\infty)$, which we have to check. For this purpose, we solve $F_a'(a,\theta) = 0$ for $\theta$ and denote the resulting expression by $\hat{\theta}(a)$. In view of Eq. (S.11), this gives

$$\hat{\theta}(a) \equiv 1 - \sqrt{\frac{av_1(a)^3v_3(a)}{(a-1)v_2(a) + v_3(a)}}, \tag{S.14}$$

with $\lim_{a\to1}\hat{\theta}(a) = 0$, $\lim_{a\to\infty}\hat{\theta}(a) = 1 - \sqrt{6}/4 \equiv \hat{\theta}_1$, and $\hat{\theta}'(a) > 0$.[19] This has the following

[19] Provided that $F_a'(a,\theta) = 0$ has a solution in $\theta$ on interval $(0,1)$, which is given by $\hat{\theta}(a)$, we have $F_{a\theta}''(a,\hat{\theta}(a)) > 0$ and $F_{aa}''(a,\hat{\theta}(a)) < 0$ – due to $\underline{a}_1(\theta) > \underline{a}_2$ and $\lim_{a\to1} F_a'(a,\theta) = -\infty$ (see above). Applying the implicit function theorem to $F_a'(a,\theta) = 0$ then establishes the positive sign of $\hat{\theta}'(a)$.

implications. If $\theta < \hat{\theta}_1$, then $F'_a(a,\theta)$ has a unique solution in $a$ on interval $(1,\infty)$, which we denote $\underline{a}_1(\theta)$ (see above). In this case, $F'_a(a,\theta) >,=,< 0$ if $a >,=,< \underline{a}_1(\theta)$. In contrast, if $\theta \geq \hat{\theta}_1$, $F'_a(a,\theta) < 0$ holds for all possible $a > 1$.

With these insights at hand, we are now prepared to determine the sign of $F(a,\theta)$. For this purpose, we can note that $\lim_{a\to 1} F(a,\theta) = \theta(2-\theta)/(1-\theta)^2 > 0$ and $\lim_{a\to\infty} F(a,\theta) = 0$, implying that $F(a,\theta) > 0$ holds for all possible finite values of $a$ if $\theta \geq \hat{\theta}_1$. In contrast, if $\theta < \hat{\theta}_1$, $F(a,\theta)$ has an extremum at $\underline{a}_1(\theta)$, which, in view of $\underline{a}_1(\theta) > \underline{a}_2$, must be a minimum. In this case, there exists a unique $\underline{a}_0(\theta) < \underline{a}_1(\theta)$, such that $F(a,\theta) >,=,< 0$ if $\underline{a}_0(\theta) >,=,< a$. Since we know from above that $f'_2(a) >,=,< 0$ is equivalent to $F(a,\theta) >,=,< 0$ and that $\lim_{a\to 1} f_2(a) = -\infty$, $\lim_{a\to\infty} f_2(a) = 0$, it is immediate that $f_2(a) < 0$ holds for all possible finite values of $a$ if $\theta \geq \hat{\theta}_1$. In contrast, if $\theta < \hat{\theta}_1$, $f_2(a)$ has an extremum at $\underline{a}_0(\theta)$, which – recollecting from above that $f'_2(a) >,=,< 0$ if $\underline{a}_0(\theta) >,=,< a$ – must be a maximum. In this case, there exists a unique $a_0(\theta) < \underline{a}_0(\theta)$, such that $f_2(a) >,=,< 0$ if $a >,=,< a_0(\theta)$. Our previous insights $f_0(1) = 0$, $f'_0(1) > 0$, and $\lim_{a\to\infty} f_0(a) < 0$ imply that $f_0(a)$ must have at least one extremum at $\hat{a}_0 > 1$, which must be a maximum (and thus requires $f_2(\hat{a}_0) < 0$). If $\theta \geq \hat{\theta}_1$, this extremum must be unique, because in this case $f_2(a) < 0$ holds for all possible finite values of $a$. If $\theta < \hat{\theta}_1$, $f_0(a)$ may have a second extremum on interval $a \in (1,\infty)$, which then must be a minimum. Both the conclusion that $f_0(a)$ cannot have more than two interior extrema and the conclusion that $f_0(a)$ has exactly one maximum follow from the properties of $f_2(a)$ and the insight that in the case of multiple extrema, a minimum must follow a maximum and vice versa. As an immediate consequence of this, it follows that there must exist a unique $\underline{a} \in (1,\infty)$, such that $f_0(a) >,=,< 0$ if $\underline{a} >,=,< a$. This completes the formal characterisation of the two properties of $f_0(a)$.

### S.3 Background material for Appendix A.10: The negative sign of $\hat{\psi}'(\hat{\eta})$

Differentiation of $\hat{\psi}(\hat{\eta})$ from Appendix A.10 establishes

$$\begin{aligned}\hat{\psi}'(\hat{\eta}) &= \frac{[1-\hat{\eta}t(1-\theta)\xi][1+\psi_2(\hat{\eta})]\psi_1(\hat{\eta}) - (1-\hat{\eta}t\xi)[1+\psi_1(\hat{\eta})]\psi_2(\hat{\eta})}{(1-\hat{\eta})[\psi_2(\hat{\eta})]^2} \\ &= \frac{1+\psi_2(\hat{\eta})}{(1-\hat{\eta})[\psi_2(\hat{\eta})]^2}\Psi(\hat{\eta};\theta,\xi), \end{aligned} \tag{S.15}$$

with

$$\Psi(\hat{\eta};\theta,\xi) \equiv (1-\hat{\eta}t\xi)[(1-\hat{\eta})^{-t\theta\xi}e^{-\hat{\eta}t\theta\xi} - 1] + \hat{\eta}t\theta\xi[(1-\hat{\eta})^{1-t\xi}e^{-\hat{\eta}t\xi} - 1], \tag{S.16}$$

and $\hat{\psi}'(\hat{\eta}) >,=,< 0$ if $\Psi(\hat{\eta};\theta,\xi) >,=,< 0$. Twice differentiating $\Psi(\cdot)$ with respect to $\theta$ (keeping $\xi$ constant) gives

$$\begin{aligned}\Psi'_\theta(\hat{\eta};\theta,\xi) &= -t\xi(1-\hat{\eta}t\xi)[\ln(1-\hat{\eta})+\hat{\eta}](1-\hat{\eta})^{-t\theta\xi}\exp[-\hat{\eta}t\xi] \\ &\qquad + \hat{\eta}t\xi\left\{(1-\hat{\eta})^{1-t\xi}\exp[-\hat{\eta}t\xi] - 1\right\}, \end{aligned} \tag{S.17}$$

$$\Psi''_{\theta\theta}(\hat{\eta};\theta,\xi) = (t\xi)^2(1-\hat{\eta}t\xi)[\ln(1-\hat{\eta})+\hat{\eta}]^2(1-\hat{\eta})^{-t\theta\xi}\exp[-\hat{\eta}t\xi] > 0. \tag{S.18}$$

Hence, if $\Psi(\hat{\eta};\theta,\xi)$ has an extremum in $\theta$, it must be a minimum. We can now compute

$$\Psi(\hat{\eta};1,\xi) = (1-\hat{\eta}t\xi)\left\{(1-\hat{\eta})^{-t\xi}\exp\left[-\hat{\eta}t\xi\right]-1\right\} + \hat{\eta}t\xi\left\{(1-\hat{\eta})^{1-t\xi}\exp\left[-\hat{\eta}t\xi\right]-1\right\}. \tag{S.19}$$

Differentiating the latter with respect to $\hat{\eta}$ gives

$$\Psi_{\hat{\eta}}(\hat{\eta};1,\xi) = t\xi\hat{\eta}(1-\hat{\eta})^{-t\xi}\exp\left[-\hat{\eta}t\xi\right]\left\{(1-\hat{\eta}t\xi)\frac{\hat{\eta}}{1-\hat{\eta}}-1\right\}, \tag{S.20}$$

which, in view of $t > 1/\xi$ (which is always fulfilled if $t > t_1$), is unambiguously negative. Since $\Psi(0;1,\xi) = 0$, we can therefore safely conclude that $\Psi(\hat{\eta};1,\xi) < 0$ holds for all $\hat{\eta} > 0$. Combining this with $\Psi(\hat{\eta};0,\xi) = 0$, it follows from $\Psi''_{\theta\theta}(\hat{\eta};\theta,\xi) > 0$ that $\Psi(\hat{\eta};\theta,\xi) < 0$ holds for all possible $\hat{\theta},\hat{\eta} > 0$, proving that $\hat{\psi}'(\hat{\eta}) < 0$.

# References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