Optimal Trade Policy with Intermediate Goods
Preliminary

Mark Razhev*
Melbourne University

October 31, 2016

Abstract

This paper studies optimal trade policy in the presence of intermediate goods in a general equilibrium setting. I consider a multi-industry two-stage production economy where both final and intermediate products are traded and the available policy instruments include all import tariffs and export taxes. Import taxes for intermediate goods are uniform, i.e. do not vary across industries despite differences in comparative advantage, trade costs or trade elasticities. Other policy instruments are heterogeneous, responding to variation in model parameters. Exact analytical expressions are provided for final import tariffs and export taxes. I find that stronger international production sharing reduces optimal import tariffs but increases optimal export taxes.

*Email: mark.razhev@unimelb.edu.au. I thank Oleg Itskhoki and Steve Redding for helpful comments and suggestions.

1 Introduction

The growth of international production sharing puts supply chain trade at the center of policy discussion.[1] Yet the theory of international trade is not ready to accommodate this shift, as argued in Blanchard, Bown, and Johnson (2016), henceforth BBJ. These authors offer one of the first systematic studies of the role of supply chains in trade policy. The present paper attempts to make further progress in this direction on the theory side.

I study optimal unilateral import tariffs and export taxes for a two-stage production structure based on Yi (2003, 2010). While quite specific and not entirely realistic, this model environment allows incorporating supply chain considerations in a fully structural way.[2] In terms of methodology, I follow Costinot, Donaldson, Vogel, and Werning (2015) who solve for optimal trade policy indirectly by formulating the problem in terms of allocations.

I consider a two-country world in which sophisticated Home maximizes domestic utility using a full set of trade taxes while Foreign is passive. Even for this simple two-stage production structure, there are rich interactions between upstream and downstream decisions. While Section 2 describes these interactions in some detail, the direct approach to characterizing optimal trade instruments yields little progress.

Section 3 then considers an alternative formulation of the policy problem in the spirit of Costinot et al. (2015). I rely on the fact that using the full set of trade taxes (both final and intermediate, import and export) provides enough flexibility to target all marginal rates of substitution in the world economy, so that the problem admits an equivalent "primal" formulation in terms of allocations. Section 3 delivers the main results of the paper.

For **import tariffs**, I find that a uniform rate is applied to foreign value content while domestic content is subsidized. In the model, intermediate goods only use labor in their production, so imported intermediates embody purely foreign labor and have a common tariff despite heterogeneity in comparative advantage, trade costs or trade elasticities. This finding mirrors the uniformity result for import tariffs in models without intermediate goods, provided in Opp (2010) and Costinot et al. (2015). It stems from the fact that, in a large class of neoclassical trade models, countries effectively exchange primary factors, in this case labor. The role of import tariffs is then to tilt the relative

[1] See, for example, Baldwin (2012).

[2] In the theory part of the paper, BBJ abstract from the actual input-output linkages and postulate a production function that depends directly on domestic and foreign value added. Although such specification may result from some kind of approximation, its validity for policy exercises is unclear.

labor price. With factor mobility across industries within countries, this mechanism operates at the economy level, so in distorting the common (across industries) wages no heterogeneity in import tariffs arises. Introducing trade in intermediate inputs does not change the core of this logic because all products are eventually produced from labor, but the uniformity result is modified in two ways. First, as final products incorporate labor services from both countries, a uniform import tariff is applied to foreign content only. Stronger international production sharing reduces the incentives to impose import protection for final goods, because some part of import spending returns to the country in the form of export revenue from intermediate goods. Although in a different framework, BBJ provide a similar result that import tariffs for final goods are decreasing in domestic value content. The theoretical result of the present paper also highlights the role of intermediate export taxes. Second, unlike in Costinot et al. (2015), this uniformity of import tariffs is no longer robust to excluding export taxes from the set of policy variables.

The determination of **export taxes** is related to the problem of a multiproduct monopolist. One major distinction for intermediate export taxes is that the Home government must also take into account productivity losses in Foreign due to higher material costs which eventually hurt Home consumers. Because of this complication, there is no elegant analytical expression for intermediate export taxes. For final export taxes, however, the optimality condition takes a particularly simple form: the standard monopoly markup is discounted by the level of the corresponding final import tariff. Heterogeneity in final export taxes is explained by variation in market power and the strength of production sharing captured by final import tariffs. While the first factor is very intuitive and is already present in Costinot et al. (2015), the second one is specific to supply chain trade. Higher export taxes induce consumer expenditure switching in Foreign, which increases Foreign final production and hence demand for intermediate goods, including import demand. The latter is beneficial for Home as it increases Foreign demand for Home labor and also export tax revenue from intermediate goods. The strength of this channel varies with the share of Home inputs in Foreign final production and the level of intermediate export taxes in a way that the level of final import tariffs is the single summary statistic. This result challenges the view that international production sharing should result in a more liberal trade policy. As a thought exercise, consider a decrease in intermediate trade costs that reduces intermediate domestic shares as countries source more inputs from their trade partners. The lower final import tariffs may then be offset by higher final export taxes, so that the impact on the overall trade flows is ambiguous.

The rest of the paper is structured as follows. Section 2 of the paper introduces the environment and discusses the role of final and intermediate trade taxes. Section 3 applies the primal approach to the government's problem and establishes the main analytical results. Section 3 also outlines several comparative statics results from simulation exercises. Section 4 concludes.

2 Direct Problem

This section introduces the environment and discusses the direct approach to solving for optimal policy.

2.1 Preferences and Technology

The world economy consists of two countries, Home and Foreign. Labor is the only primary factor of production. It is supplied inelastically and its endowments are denoted with L and L^* (the Foreign variables have asterisk notation). Different products are indexed by $i = 1..N$. Each good is produced in two stages: stage one uses labor only while stage two uses labor and stage one output. Throughout the paper, "stage one" is used interchangeably with "intermediate" or "upstream" and "stage two" with "final" or "downstream". Only final products are consumed and preferences, similar in both countries, are given by the log function

$$U = \sum_{i=1}^{N} \alpha_i \log C_i, \tag{2.1}$$

with expenditure shares $\alpha_i > 0$, $\sum_i \alpha_i = 1$.

The technology for intermediate goods is linear in labor inputs l_i^I with unit requirements Z_i^I:

$$q_i^I = \frac{1}{Z_i^I} l_i^I. \tag{2.2}$$

The technology for final goods is Cobb-Douglas constant returns to scale in labor inputs l_i^F and intermediate inputs X_i with inverse productivity Z_i^F:

$$q_i^F = \frac{1}{Z_i^F} \left(\frac{l_i^F}{1-\beta_i} \right)^{1-\beta_i} \left(\frac{X_i}{\beta_i} \right)^{\beta_i}, \tag{2.3}$$

where $0 \leq \beta_i < 1$ is the intermediate input share. It is the same for both countries but can vary across industries.

For international trade, the standard iceberg trade cost specification is adopted: τ_i^I (τ_i^F) units of an intermediate (final) product must be shipped from one country in order for one unit to arrive in the other country. Trade costs are assumed to be symmetric for expositional simplicity, although this assumption does not affect the results. Domestic trade costs are absent.

Our description of the environment ends with the Armington product differentiation assumption. First, each final good i enters consumption (2.1) as a CES bundle C_i which aggregates the corresponding domestic and imported varieties c_{id} and c_{im} with the elasticity of substitution $\rho_i^F > 1$:

$$C_i = \left(c_{id}^{\frac{\rho_i^F - 1}{\rho_i^F}} + c_{im}^{\frac{\rho_i^F - 1}{\rho_i^F}} \right)^{\frac{\rho_i^F}{\rho_i^F - 1}}. \tag{2.4}$$

Second, each intermediate good i enters production (2.3) as a CES bundle X_i which aggregates the corresponding domestic and imported varieties x_{id} and x_{im} with the elasticity of substitution $\rho_i^I > 1$:

$$X_i = \left(x_{id}^{\frac{\rho_i^I - 1}{\rho_i^I}} + x_{im}^{\frac{\rho_i^I - 1}{\rho_i^I}} \right)^{\frac{\rho_i^I}{\rho_i^I - 1}}. \tag{2.5}$$

The parameters ρ_i^F and ρ_i^I are the same in Home and Foreign but can vary across industries. Introducing the Armington love for variety is a departure from the original two-stage specification in Yi (2003, 2010) which only features Ricardian technological heterogeneity. This is primarily driven by tractability concerns[3] but is also justified on empirical grounds as strong intra-industry trade is present for both final and intermediate goods.

2.2 Home Economy

Consumers and firms in both countries behave competitively, so all prices of domestically traded goods are equal to marginal costs. There is a government in Home that sets trade taxes and redistributes the revenue to Home consumers. I first characterize consumption and production decisions conditional on policy instruments. Import taxes for final and intermediate products are introduced as coefficients and denoted with tm_i^F and tm_i^I. Thus,

[3] A finite Armington elasticity generates smooth reactions of trade flows to changes in relative costs. There is always positive production and two-way trade for all products and the problem is well-behaved even with a finite number of industries.

in case of final goods, Home consumers face a price that is $(tm_i^F - 1) \cdot 100\%$ higher than $\tau_i^F MC_i^{F*}$ which is Foreign marginal costs multiplied by the iceberg trade costs. Export taxes (also coefficients) are denoted tx_i^F and tx_i^I. For final goods, Home firms receive a fraction $\frac{1}{tx_i^F}$ of a price paid by Foreign consumers. In other words, with competitive pricing Home firms charge Foreign consumers $tx_i^F \tau_i^F MC_i^F$, or export prices are $(tx_i^F - 1) \cdot 100\%$ higher than $\tau_i^F MC_i^F$.

Marginal costs

With W denoting the Home wage and according to (2.2) and (2.3), stage-one marginal costs are

$$MC_i^I = Z_i^I W \tag{2.6}$$

and stage-two marginal costs are

$$MC_i^F = Z_i^F W^{1-\beta_i} (P_{Mi})^{\beta_i}, \tag{2.7}$$

where P_{Mi} is the price index for material input in (2.3). With the CES specification (2.5), it is equal to

$$P_{Mi} = \left(\left(MC_i^I \right)^{1-\rho_i^I} + \left(tm_i^I \tau_i^I MC_i^{I*} \right)^{1-\rho_i^I} \right)^{\frac{1}{1-\rho_i^I}}. \tag{2.8}$$

Note that stage-two Home firms import intermediates paying $tm_i^I \tau_i^I$ times Foreign marginal costs, so upstream import tariffs (just as trade costs) make downstream Home firms less productive.

Consumer demand

The nominal income of Home consumers consists of labor income WL and tax revenue R which is specified later. Maximizing (2.1), the amount $\alpha_i (WL + R)$ is spent on final product i. Under the Armington preferences (2.4) we find that a fraction λ_i^F of this expenditure is allocated to the domestic variety and $1 - \lambda_i^F$ to the imported variety, with

$$\lambda_i^F = \frac{\left(MC_i^F \right)^{1-\rho_i^F}}{\left(MC_i^F \right)^{1-\rho_i^F} + \left(tm_i^F \tau_i^F MC_i^{F*} \right)^{1-\rho_i^F}}. \tag{2.9}$$

The quantities demanded by Home consumers are then

$$c_{id} = \frac{\alpha_i \lambda_i^F (WL + R)}{MC_i^F} \quad \text{and} \quad c_{im} = \frac{\alpha_i \left(1 - \lambda_i^F\right) (WL + R)}{tm_i^F \tau_i^F MC_i^{F*}}. \tag{2.10}$$

Firm demand

The final production of industry i in Home satisfies domestic demand c_{id} and Foreign import demand c_{im}^* adjusted by trade costs τ_i^F. Therefore, the revenue of final producers in Home, denoted y_i^F, is

$$y_i^F = MC_i^F \cdot \left(c_{id} + \tau_i^F c_{im}^*\right). \tag{2.11}$$

According to (2.3), final producers in industry i spend $(1-\beta_i)\, y_i^F$ on labor, so they employ

$$l_i^F = \frac{(1-\beta_i)\, y_i^F}{W} \text{ workers.} \tag{2.12}$$

Also according to (2.3), the amount $\beta_i y_i^F$ is spent on intermediate goods and there is again a split of this expenditure between domestic and imported varieties in (2.4) with the domestic share given by

$$\lambda_i^I = \frac{\left(MC_i^I\right)^{1-\rho_i^I}}{\left(MC_i^I\right)^{1-\rho_i^I} + \left(tm_i^I \tau_i^I MC_i^{I*}\right)^{1-\rho_i^I}}. \tag{2.13}$$

Therefore, the quantities demanded by intermediate producers are

$$x_{id} = \frac{\lambda_i^I \beta_i y_i^F}{MC_i^I} \text{ and } x_{im} = \frac{\left(1-\lambda_i^I\right)\beta_i y_i^F}{tm_i^I \tau_i^I MC_i^{I*}}. \tag{2.14}$$

Next, the revenue of intermediate producers, y_i^I, comes from serving domestic demand x_{id} and Foreign import demand x_{im}^* adjusted by trade costs τ_i^I:

$$y_i^I = MC_i^I \cdot \left(x_{id} + \tau_i^I x_{im}^*\right). \tag{2.15}$$

This amount is entirely spent on labor, so intermediate producers in industry i employ

$$l_i^I = \frac{y_i^I}{W} \text{ workers.} \tag{2.16}$$

Tax revenue

As Home consumers spend $\left(1-\lambda_i^F\right)\alpha_i\,(WL+R)$ on importing final product i, Foreign firms get a fraction $\frac{1}{tm_i^F}$ of that amount. That means that the rest, a fraction $\frac{tm_i^F-1}{tm_i^F}$, is received by the Home government. Similarly, Foreign consumers purchase the value $\left(1-\lambda_i^{F*}\right)\alpha_i W^* L^*$ of Home goods, and it is split between Home firms and the government with the shares $\frac{1}{tx_i^F}$ and $\frac{tx_i^F-1}{tx_i^F}$. The tax revenue arising from intermediate trade is analogous, with consumer spending replaced by firm expenditure on materials $\beta_i y_i^F$ and

$\beta_i y_i^{F*}$ (only downstream producers use intermediates). Combining the pieces, the total tax revenue is

$$R = \sum_i \frac{tm_i^F - 1}{tm_i^F}\left(1-\lambda_i^F\right)\alpha_i\left(WL+R\right) + \sum_i \frac{tm_i^I - 1}{tm_i^I}\left(1-\lambda_i^I\right)\beta_i y_i^F + \tag{2.17}$$

$$+\sum_i \frac{tx_i^F - 1}{tx_i^F}\left(1-\lambda_i^{F*}\right)\alpha_i W^* L^* + \sum_i \frac{tx_i^I - 1}{tx_i^I}\left(1-\lambda_i^{I*}\right)\beta_i y_i^{F*}.$$

Market clearing

Note that goods market clearing is already incorporated in (2.11) and (2.15). The condition for labor market clearing is

$$\sum_i \left(l_i^I + l_i^F\right) = L. \tag{2.18}$$

Note also that trade balance is implied by market clearing and the fact that Home consumers balance their budget spending $WL+R$.

2.3 Foreign Economy

All equilibrium relationships for Foreign are exactly similar. For completeness, I provide them in Part I of Appendix A.1 without detailed explanation.

2.4 Trade Policy

Domestic shares

Before formulating the policy problem, it is convenient to minimize the involved notation. In this section, the policy problem has $4 \times N + 2$ variables which are trade taxes $\left(tm_i^F, tm_i^I, tx_i^F, tx_i^I\right)_{i=1}^N$ and wages W and W^*, as the planner takes into account the general equilibrium effects on wages. (In fact, as discussed below, both wages can be normalized as long as the complete set of export and import taxes is available.) All production and consumption decisions can be expressed as explicit functions of these variables, so in this formulation the problem only has two constraints which are Home and Foreign labor market clearing conditions. The problem's presentation simplifies significantly once we recognize that both consumer welfare and firm labor demand can be expressed through the domestic shares together with wages and taxes.

Table 1 lists all domestic shares as explicit functions of trade taxes and wages with pluses and minuses indicating the signs of the corresponding partial derivatives. It simply summarizes the equations (2.6)-(2.9) and (2.13) for Home as well as (A.1)-(A.4) and (A.8) for Foreign.

Table 1

Intermediate domestic share in Home	$\lambda_i^I\left(\underset{+}{tm_i^I}, \underset{-}{W}, \underset{+}{W^*}\right)$
Intermediate domestic share in Foreign	$\lambda_i^{I*}\left(\underset{+}{tx_i^I}, \underset{+}{W}, \underset{-}{W^*}\right)$
Final domestic share in Home	$\lambda_i^F\left(\underset{+}{tm_i^F}, \underset{-}{W}, \underset{+}{W^*}, \underset{-}{tm_i^I}, \underset{+}{tx_i^I}\right)$
Final domestic share in Foreign	$\lambda_i^{F*}\left(\underset{+}{tx_i^F}, \underset{+}{W}, \underset{-}{W^*}, \underset{+}{tm_i^I}, \underset{-}{tx_i^I}\right)$

Naturally, intermediate import tariffs tm_i^I reduce Home import of materials, so λ_i^I is increasing in tm_i^I. The impact of intermediate tariffs also propagates downstream. As the material price index (2.8) can be expressed as $P_{Mi} = Z_i^I W \left(\lambda_i^I\right)^{\frac{1}{\rho_i^I - 1}}$ and therefore Home marginal costs increase in tm_i^I, Home loses competitiveness in final goods, so consumers switch to Foreign products: λ_i^F is decreasing and λ_i^{F*} is increasing in tm_i^I.

The direct impact of intermediate export taxes tx_i^I is to reduce Foreign imports in downstream: λ_i^{I*} is increasing. In addition to this, as a consequence of higher material costs, Foreign final firms become less competitive, so λ_i^F is increasing and λ_i^{F*} is decreasing in tx_i^I.

The effect of final trade taxes tm_i^F and tx_i^F is simply to reduce final imports in Home and Foreign respectively: λ_i^F is increasing in tm_i^F and λ_i^{F*} is increasing in tx_i^F.

Finally, higher labor prices reduce competitiveness in both upstream and downstream, so all λ's are decreasing in domestic wages and increasing in trade partners' wages.

Objective function

As the Home government sets trade taxes to maximize consumer utility, we first combine some of the above relationships to solve for equilibrium welfare. The log price index for utility (2.1) equals (up to a constant depending on α's)

$$\log P = \sum\nolimits_i \alpha_i \log P_i, \tag{2.19}$$

with the industry-level price indices consistent with the Armington aggregator (2.4)

$$P_i = \left(\left(MC_i^F\right)^{1-\rho_i^F} + \left(tm_i^F \tau_i^F MC_i^{F*}\right)^{1-\rho_i^F}\right)^{\frac{1}{1-\rho_i^F}}. \tag{2.20}$$

The objective function for the government can therefore be written as

$$U = \log(WL + R) - \sum_i \alpha_i \log P_i. \tag{2.21}$$

Connecting to the domestic shares, we can substitute $P_{Mi} = Z_i^I W \left(\lambda_i^I\right)^{\frac{1}{\rho_i^I - 1}}$ into downstream marginal costs (2.7) and then use $P_i = MC_i^F \cdot \left(\lambda_i^F\right)^{\frac{1}{\rho_i^F - 1}}$ to write (2.20) as

$$P_i = W \cdot Z_i^F \left(Z_i^I\right)^{\beta_i} \cdot \left(\lambda_i^I\right)^{\frac{\beta_i}{\rho_i^I - 1}} \cdot \left(\lambda_i^F\right)^{\frac{1}{\rho_i^F - 1}}. \tag{2.22}$$

From this expression we can see the partial equilibrium effect of trade taxes on consumer prices. Import tariffs for both final and intermediate goods increase the price index (2.22). Note that the impact of upstream import tariffs is mitigated by consumer switching to Foreign final goods ($tm_i^I \uparrow \rightarrow \lambda_i^I \uparrow$, $\lambda_i^F \downarrow$), but this is not enough to overcome the direct effect of higher material costs.[4] In addition to this, intermediate export taxes also increase P_i by raising final Foreign costs ($tx_i^I \uparrow \rightarrow MC_i^{F*} \uparrow \rightarrow \lambda_i^F \uparrow$).

Constraints

First, we obtain firm revenue expressed in terms of domestic shares. Combining (2.11) and (2.10) as well as the corresponding equations for Foreign, the revenue of Home final firms is

$$y_i^F = \lambda_i^F \alpha_i (WL + R) + \frac{1}{tx_i^F} \left(1 - \lambda_i^{F*}\right) \alpha_i W^* L^*, \tag{2.23}$$

and for Foreign final producers it is

$$y_i^{F*} = \lambda_i^{F*} \alpha_i W^* L^* + \frac{1}{tm_i^F} \left(1 - \lambda_i^F\right) \alpha_i (WL + R). \tag{2.24}$$

For intermediate producers, combining (2.15) and (2.14) yields

$$y_i^I = \lambda_i^I \beta_i y_i^F + \frac{1}{tx_i^I} \left(1 - \lambda_i^{I*}\right) \beta_i y_i^{F*} \text{ and} \tag{2.25}$$

and for Foreign final producers we have

[4] Formally, tm_i^I enters downstream domestic shares by shifting final Home marginal costs by the factor $\left(\lambda_i^I\right)^{\frac{\beta_i}{\rho_i^I - 1}}$. Denoting this expression ξ, the log of P_i in (2.22) is equal up to a constant to $\log \xi - \frac{1}{\rho_i^F - 1} \log\left(1 + (\gamma\xi)^{\rho_i^F - 1}\right)$, where γ is another constant. The derivative of this expression with respect to ξ is positive. In economic terms, this simply means that the price of the Armington bundle increases with the price of domestic variety as long as some amount of it is still consumed.

$$y_i^{I*} = \lambda_i^{I*}\beta_i y_i^{F*} + \frac{1}{tm_i^I}\left(1-\lambda_i^I\right)\beta_i y_i^F. \tag{2.26}$$

We can now combine these four expressions with labor demands (2.12) and (2.16) to write the labor market clearing conditions as

$$\sum\nolimits_i \left(y_i^I + (1-\beta_i)\, y_i^F\right) = WL \text{ and } \sum\nolimits_i \left(y_i^{I*} + (1-\beta_i)\, y_i^{F*}\right) = W^* L^*. \tag{2.27}$$

As for the tax revenue that enters both the objective function (2.21) and the above labor market clearing conditions, it is also expressed through the domestic shares as we already see in (2.17). To have an explicit expression, we just solve this equation which is linear in R.[5]

Without taking into account the tax revenue effect,[6] final import tariffs increase firm revenue (and hence labor demand) in Home, decrease downstream firm revenue in Foreign and increase upstream firm revenue in Foreign. As seen from (2.23)-(2.26), $tm_i^F \uparrow\rightarrow \lambda_i^F \uparrow\rightarrow y_i^F \uparrow$, $y_i^{F*} \downarrow\rightarrow y_i^I \uparrow$, $y_i^{I*} \uparrow$. (Increasing Home final production raises Home intermediate demand for both Home-produced and Foreign-produced inputs.) In the case of intermediate import tariffs, due to increasing material costs in Home we have $tm_i^I \uparrow\rightarrow \lambda_i^F \downarrow, \lambda_i^{F*} \uparrow \rightarrow y_i^F \downarrow, y_i^{F*} \uparrow$. The impact on intermediate and total labor demand is ambiguous, however. On the one hand, tm_i^I increases λ_i^I and hence y_i^I for a given value of downstream production. But strong enough consumer switching in final goods may cause Home final output to fall so much that upstream demand for labor falls as well.[7]

Final export taxes increase labor demand in Foreign, reduce downstream labor demand in Home and increase upstream labor demand in Home: $tx_i^F \uparrow\rightarrow \lambda_i^{F*} \uparrow \rightarrow y_i^F \downarrow, y_i^{F*} \uparrow\rightarrow y_i^I \uparrow, y_i^{I*} \uparrow$. Intermediate export taxes increase Foreign production of intermediates for a given level of final demand. But, again, consumer switching due to higher final costs in Foreign decreases final labor demand in Foreign and may as well reduce its total labor demand.

Normalization

[5] As an explicit function of trade taxes and wages, R looks the same as the right hand side of (2.17) without R itself adjusted by some multiplier. This is due to the income-expenditure loop as tax revenue returned to households generates new expenditure and new tax revenue.

[6] A way to think about it is to consider a large number of industries so that each one has a small share in the total tax revenue.

[7] This final expenditure switching effect is particularly strong if consumers view domestic and imported varieties as close substitutes, ρ_i^F is high.

As usual, one nominal variable can be normalized by choosing the numeraire and we put $W^* = 1$. However, there is an additional degree of freedom because once a given W and $\left(tm_i^F, tm_i^I, tx_i^F, tx_i^I\right)_{i=1}^N$ satisfy (2.27), other combinations of these variables can be constructed that also satisfy (2.27) such that the value of (2.21) stays the same. This is because the actual terms of trade vary with the relative wage adjusted by trade taxes. Let us double the Home wage and import tariffs and simultaneously reduce export taxes in half. As a result, none of the domestic shares changes because they depend on relative prices of domestic vs. imported varieties. For intermediate goods, for example, MC_i^I moves proportionally with W, MC_i^{I*} is unaffected, and so the ratio $\frac{tm_i^I \tau_i^I MC_i^{I*}}{MC_i^I}$ remains constant, giving the same λ_i^I. For Foreign, λ_i^{I*} is also unaffected as the ratio $\frac{MC_i^{I*}}{tx_i^I \tau_i^I MC_i^I}$ stays constant (higher MC_i^I is offset by lower tx_i^I). Once the domestic shares are not affected, it is straightforward to see that labor demands and real income remain constant as well. For this reason we put $W = 1$ as an additional normalization. Note that we can only do so provided that the government is equipped with the complete set of trade taxes.

Policy problem

With the chosen normalization $W = 1$ and $W^* = 1$, the Home government sets trade taxes $\left(tm_i^F, tm_i^I, tx_i^F, tx_i^I\right)_{i=1}^N$ to maximize equilibrium consumer utility (2.21) subject to the two labor market clearing conditions (2.27).[8]

The task of understanding the nature of optimal trade taxes and their variation across industries is complicated for two reasons. One is the presence of general equilibrium interactions as trade taxes in particular industries affect the economy-level labor constraints, both by changing the domestic shares and affecting the tax revenue. The other reason, specific to the present paper, is the upstream-downstream interactions in this economy. As discussed above, the effects of intermediate taxes propagate forward by the cost mechanism, and the effects of final taxes propagate backward by affecting intermediate demand.

We may try to avoid complications arising because of the tax revenue by changing the set of policy variables. The government may treat R as an independent policy variable, together with $\left(tm_i^F, tm_i^I, tx_i^F, tx_i^I\right)_{i=1}^N$, and maximize (2.21) subject to the same constraints (2.27). No additional constraints are needed because, by Walras law-type intuition, labor market clearing (2.27) implies (2.17) which is essentially the government budget balance.[9] Having more policy variables is potentially helpful for obtaining partial equilibrium in-

[8] By now we have demonstrated that all endogenous variables that appear in (2.21) and (2.27) are explicit functions of trade taxes, so there are only two constraints in the policy problem in this section.

[9] The argument, of course, relies on the fact that firm revenues in (2.27) incorporate goods market clearing and consumer budget balance.

tuition because we are able to read optimality relationships conditional on certain (in particular, general equilibrium) variables. But in the presence of forward and backward linkages this extension of policy variables does not result in a major simplification.

This technical complication motivates using the primal approach in the next section. We would like to condition optimality equations not only on general equilibrium variables but also, when possible, on upstream and downstream ones. In this case, without explaining the level of trade taxes, we may be able to understand their variation across industries and also the role of supply chains.

3 Indirect Approach

Inspired by Costinot et al. (2015), this section explores an alternative approach to the government's problem which is based on solving for optimal allocations rather than optimal values of policy instruments. We rely on the fact that using the full set of trade taxes $\left(tm_i^F, tm_i^I, tx_i^F, tx_i^I\right)_{i=1}^N$ provides enough flexibility to target all marginal rates of substitution in the economy, so that the policy problem admits an equivalent "primal" formulation in terms of allocations. More precisely, the Home government directly controls domestic allocations and sets export prices while taking into account Foreign equilibrium relationships.[10] For both intermediate and final goods, we will be able to determine the relative use of domestic vs. imported varieties in the planner's allocation and relate it to the relative marginal costs in a decentralized equilibrium. For final goods, for example, the equilibrium condition

$$\left(\frac{c_{id}}{c_{im}}\right)^{-1/\rho_i^F} = \left(\frac{\lambda_i^F}{1-\lambda_i^F}\right)^{\frac{1}{1-\rho_i^F}} = \frac{1}{tm_i^F}\frac{MC_i^F}{MC_i^{F*}\tau_i^F} \tag{3.1}$$

allows to infer tariffs tm_i^F from the knowledge of $\frac{c_{id}}{c_{im}}$ and $\frac{MC_i^F}{MC_i^{F*}\tau_i^F}$. Note that our focus is still on the government's problem of setting optimal trade taxes, so we only use the primal formulation to get key intermediate results and then return to the decentralized formulation of Section 2.[11]

We adopt a normalization by setting the Foreign wage $W^* = 1$. When connecting to a decentralized equilibrium, the Home wage is also normalized as $W = 1$. The Home

[10] Perhaps a better term for this formulation would be "semiprimal approach". In principle, the problem can be set in terms of allocations only, so that export prices are also recovered through wedges rather than solved for directly. I prefer the mixed setup of this section as it involves less algebra.

[11] In particular, alternative implementations of the optimal allocation (such as involving production and consumption taxes) are not discussed.

government directly controls production and consumption decisions in Home, setting the variables $\left(c_{id}, c_{im}, x_{id}, x_{im}, X_i, l_i^I, l_i^F\right)$. In addition to this, it sets export prices at which Foreign firms and consumers purchase Home-produced goods. These prices, for intermediate and final products, are denoted Px_i^I and Px_i^F. The planner's problem has therefore $9 \times N$ controls, while the Foreign variables that appear in the constraints are functions of these controls.

Foreign equilibrium

Part II of Appendix Appendix A.1 derives the relevant Foreign variables as functions of the controls. First, the final and intermediate domestic shares are

$$\lambda_i^{I*}\left(Px_i^I\right) = \frac{\left(Z_i^{I*}\right)^{1-\rho_i^I}}{\left(Z_i^{I*}\right)^{1-\rho_i^I} + \left(Px_i^I\right)^{1-\rho_i^I}} \text{ and} \tag{3.2}$$

$$\lambda_i^{F*}\left(Px_i^I, Px_i^F\right) = \frac{\left(Z_i^{F*}\left(Z_i^{I*}\right)^{\beta_i}\left(\lambda_i^{I*}\right)^{\frac{\beta_i}{\rho_i^I-1}}\right)^{1-\rho_i^F}}{\left(Z_i^{F*}\left(Z_i^{I*}\right)^{\beta_i}\left(\lambda_i^{I*}\right)^{\frac{\beta_i}{\rho_i^I-1}}\right)^{1-\rho_i^F} + \left(Px_i^F\right)^{1-\rho_i^F}}. \tag{3.3}$$

The sales of final and intermediate producers are

$$y_i^{F*}\left(Px_i^I, Px_i^F, c_{im}\right) = \alpha_i \lambda_i^{F*} L^* + \underbrace{Z_i^{F*}\left(Z_i^{I*}\right)^{\beta_i}\left(\lambda_i^{I*}\right)^{\frac{\beta_i}{\rho_i^F-1}}}_{MC_i^{F*}} \tau_i^F c_{im} \text{ and} \tag{3.4}$$

$$y_i^{I*}\left(Px_i^I, Px_i^F, c_{im}, x_{im}\right) = \lambda_i^{I*}\beta_i y_i^{F*} + Z_i^{I*}\tau_i^I x_{im}. \tag{3.5}$$

The import quantities demanded by consumers and intermediate producers are

$$c_{im}^*\left(Px_i^I, Px_i^F\right) = \frac{\alpha_i\left(1-\lambda_i^{F*}\right)L^*}{Px_i^F} \text{ and } x_{im}^*\left(Px_i^I, Px_i^F, c_{im}\right) = \frac{\left(1-\lambda_i^{I*}\right)\beta_i y_i^{F*}}{Px_i^I}. \tag{3.6}$$

Finally, employment is equal or proportional to firm sales:

$$l_i^{I*}\left(Px_i^I, Px_i^F, c_{im}, x_{im}\right) = y_i^{I*} \text{ and } l_i^{F*}\left(Px_i^I, Px_i^F, c_{im}\right) = \left(1-\beta_i\right) y_i^{F*}. \tag{3.7}$$

Home policy problem

The Home government maximizes domestic utility subject to technological and resource constraints:

$$\max_{c_{id},c_{im},x_{id},x_{im},X_i,l_i^I,l_i^F,Px_i^I,Px_i^F} U = \sum_i \alpha_i \log\left(c_{id}^{1-1/\rho_i^F} + c_{im}^{1-1/\rho_i^F}\right)^{\frac{\rho_i^F}{1-\rho_i^F}} \text{ subject to} \tag{3.8}$$

$$\text{Final Home output:}\quad c_{id} + \tau_i^F c_{im}^*\left(Px_i^I,Px_i^F\right) = \frac{1}{Z_i^F}\left(\frac{l_i^F}{1-\beta_i}\right)^{1-\beta_i}\left(\frac{X_i}{\beta_i}\right)^{\beta_i} \quad |\mu_i^F \tag{3.9}$$

$$\text{Materials Armington:}\quad X_i = \left(x_{id}^{1-1/\rho_i^I} + x_{im}^{1-1/\rho_i^I}\right)^{\frac{\rho_i^I}{1-\rho_i^I}} \quad |\chi_i \tag{3.10}$$

$$\text{Intermediate Home output:}\quad x_{id} + \tau_i^I x_{im}^*\left(Px_i^I,Px_i^F,c_{im}\right) = \frac{1}{Z_i^I} l_i^I \quad |\mu_i^I \tag{3.11}$$

$$\text{Home labor constraint:}\quad \sum_i \left(l_i^I + l_i^F\right) = L \quad |\overline{\mu} \tag{3.12}$$

$$\text{Foreign labor constraint:}\quad \sum_i \left[l_i^{I*}\left(Px_i^I,Px_i^F,c_{im},x_{im}\right) + l_i^{F*}\left(Px_i^I,Px_i^F,c_{im}\right)\right] = L^* \ |\overline{\mu}^* \tag{3.13}$$

For simplicity, I introduce all constraints as equalities.[12] Note also that we keep the aggregate materials input X_i as a separate control and not put x_{id} and x_{im} directly into (3.9). This is slightly more convenient in terms of exposition.

After forming the Lagrangian as

$$\Lambda = \sum_i \alpha_i \log\left(c_{id}^{1-1/\rho_i^F} + c_{im}^{1-1/\rho_i^F}\right)^{\frac{\rho_i^F}{1-\rho_i^F}} + \sum_i \chi_i\left[\left(x_{id}^{1-1/\rho_i^I} + x_{im}^{1-1/\rho_i^I}\right)^{\frac{\rho_i^I}{1-\rho_i^I}} - X_i\right]$$
$$+\sum_i \mu_i^F\left[\frac{1}{Z_i^F}\left(\frac{l_i^F}{1-\beta_i}\right)^{1-\beta_i}\left(\frac{X_i}{\beta_i}\right)^{\beta_i} - c_{id} - \tau_i^F c_{im}^*\right] + \sum_i \mu_i^I\left[\frac{1}{Z_i^I} l_i^I - x_{id} - \tau_i^I x_{im}^*\right]$$
$$+\overline{\mu}\left[L - \sum_i \left(l_i^I + l_i^F\right)\right] + \overline{\mu}^*\left[L^* - \sum_i \left(l_i^{I*} + l_i^{F*}\right)\right] \tag{3.14}$$

we are ready to characterize optimal policy.

[12] In principle, the Home planner might not use all Home labor or produce more goods than used in consumption or production, but that would never be optimal.

3.1 Import tariffs

Intermediate goods

By differentiating (3.14) with respect to domestic and imported inputs x_{id} and x_{im} and taking the ratio, we arrive at

$$\left(\frac{x_{id}}{x_{im}}\right)^{-1/\rho_i^I} = \frac{\mu_i^I}{\overline{\mu}^* Z_i^{I*} \tau_i^I} = \frac{\overline{\mu}}{\overline{\mu}^*} \frac{Z_i^I}{Z_i^{I*} \tau_i^I}. \tag{3.15}$$

The relative quantity of intermediate domestic variety is inversely related to the relative labor costs $\frac{Z_i^I}{Z_i^{I*}\tau_i^I}$ adjusted by a common (across industries) factor $\frac{\overline{\mu}}{\overline{\mu}^*}$ which is the relative shadow price of labor. This expression is particularly simple because only labor is used to produce intermediate goods, so the relative costs of providing x_{id} and x_{im} are directly linked to exogenous labor requirements. From (3.15) we immediately establish the pattern of variation in intermediate import tariffs. As $\left(\frac{x_{id}}{x_{im}}\right)^{-1/\rho_i^I} = \left(\frac{\lambda_i^I}{1-\lambda_i^I}\right)^{\frac{1}{1-\rho_i^I}} = \frac{MC_i^I}{tm_i^I \tau_i^I MC_i^{I*}}$ in a market equilibrium and the upstream marginal costs are simply unit labor requirements,[13] we conclude that

$$\frac{\overline{\mu}}{\overline{\mu}^*} \frac{Z_i^I}{Z_i^{I*} \tau_i^I} = \frac{Z_i^I}{tm_i^I Z_i^{I*} \tau_i^I} \Rightarrow tm_i^I = \frac{\overline{\mu}^*}{\overline{\mu}} \text{ for all } i = 1..N. \tag{3.16}$$

While this condition is not informative of the *level* of import tariffs, it gives a sharp result about the *variation* across industries. Namely, import tariffs for intermediate goods are uniform despite the presence of heterogeneity in labor costs, trade costs, or elasticities of substitution. Thus we have proven the following

Proposition 3.1. *Optimal import tariffs for intermediate goods are uniform: $tm_i^I = tm^I$ for all $i = 1..N$.*

This result is similar to Costinot et al. (2015). At the heart of it is the fact that countries effectively exchange labor while trading goods and the role of import tariffs is to tilt the relative price of labor. With factor mobility across industries within countries, this mechanism operates at the economy level, so in distorting the common (across industries) wages no heterogeneity in import tariffs arises. Introducing trade in intermediate inputs does not change the core of this logic because all products are eventually produced from labor. However, as we will see below, the uniformity result generalizes so that a common rate is applied to the Foreign labor content rather than the entire product value. From now on we suppress the industry subscript in tm_i^I.

[13] Now we also apply our normalization $W^* = W = 1$.

Final goods

We proceed by differentiating (3.14) with respect to domestic and imported final products c_{id} and c_{im}. This is more complicated compared to intermediate goods because Home import c_{im} affects not only downstream Foreign employment $l_i^{F^*}$ but also its upstream employment l_i^{I*}, and it also enters the intermediate Home output constraint (3.11). The latter is because Home import demand for final goods affects Foreign intermediate import demand. While the first order condition for c_{id} is simply $\frac{\partial U}{\partial c_{id}} = \mu_i^F$, for c_{im} we have

$$\frac{\partial U}{\partial c_{im}} = \overline{\mu}^* \left[\frac{\partial l_i^{F^*}}{\partial c_{im}} + \frac{\partial l_i^{I^*}}{\partial c_{im}} \right] + \mu_i^I \tau_i^I \frac{\partial x_{im}^*}{\partial c_{im}}. \tag{3.17}$$

This difference is due to the fact that the Home government must explicitly take into account the Foreign supply chain structure, namely upstream propagation of demand. In contrast, maximization with respect to domestic final consumption c_{id} is conditional on the use of intermediate inputs in Home. Of course, the choices c_{im} and intermediate inputs in Home must still be consistent as the constraint (3.9) requires, but this linkage is accounted for by the Lagrange multiplier μ_i^F. From (3.4)-(3.5) and (3.7), $\frac{\partial l_i^{F^*}}{\partial c_{im}} = (1 - \beta_i) MC_i^{F*} \tau_i^F$ and $\frac{\partial l_i^{I^*}}{\partial c_{im}} = \lambda_i^{I*} \beta_i MC_i^{F*} \tau_i^F$. Also, $\frac{\partial x_{im}^*}{\partial c_{im}} = \frac{(1-\lambda_i^{I*})\beta_i}{Px_i^I} \frac{\partial y_i^{F*}}{\partial c_{im}} = \frac{(1-\lambda_i^{I*})\beta_i}{Px_i^I} MC_i^{F*} \tau_i^F$. Combining the pieces, (3.17) becomes

$$\frac{\partial U}{\partial c_{im}} = \overline{\mu}^* MC_i^{F*} \tau_i^F \left[1 - \beta_i \left(1 - \lambda_i^{I*}\right) \left(1 - \frac{\mu_i^I \tau_i^I}{\overline{\mu}^* Px_i^I}\right) \right]. \tag{3.18}$$

We further observe that $\mu_i^I = \overline{\mu} Z_i^I$ and use the connection between the direct and indirect formulations of the government's problem.[14] The ratio $\frac{\mu_i^I \tau_i^I}{\overline{\mu}^* Px_i^I}$ in (3.18) can be expressed as $\frac{\overline{\mu}}{\overline{\mu}^*} \frac{Z_i^I \tau_i^I}{Px_i^I} = \frac{1}{tm^I tx_i^I}$, after which the ratio $\frac{\partial U}{\partial c_{id}} / \frac{\partial U}{\partial c_{im}}$ is

$$\left(\frac{c_{id}}{c_{im}} \right)^{-1/\rho_i^F} = \left(\frac{\lambda_i^F}{1 - \lambda_i^F} \right)^{\frac{1}{1-\rho_i^F}} = \frac{1}{tm^I - \beta_i \left(1 - \lambda_i^{I*}\right) \left(tm^I - \frac{1}{tx_i^I} \right)} \frac{MC_i^F}{MC_i^{F*} \tau_i^F}. \tag{3.19}$$

This expression determines final import tariffs tm_i^F as the wedge $tm^I - \beta_i \left(1 - \lambda_i^{I*}\right) \left(tm^I - \frac{1}{tx_i^I} \right)$. Expressing this wedge as $\left(1 - \beta_i + \beta_i \lambda_i^{I*}\right) tm^I + \beta_i \left(1 - \lambda_i^{I*}\right) \frac{1}{tx_i^I}$ completes the proof of the following

[14] In particular, the export price Px_i^I is equal to $MC_i^I \tau_i^I$ times the tax coefficient tx_i^I, or $Px_i^I = Z_i^I \tau_i^I tx_i^I$. Also, with $W = 1$ the optimal value of μ_i^F is related to decentralized marginal costs as $\mu_i^F = \overline{\mu} MC_i^F$.

Proposition 3.2. *Optimal import tariffs for final goods satisfy*

$$tm_i^F = \left(1 - \beta_i + \beta_i \lambda_i^{I*}\right) tm^I + \beta_i \left(1 - \lambda_i^{I*}\right) \frac{1}{tx_i^I}. \tag{3.20}$$

According to this proposition, variation in final import tariffs arises when Foreign final industries differentially use Home-produced intermediates, or due to variation in intermediate export taxes tx_i^I. The term $\left(1 - \beta_i + \beta_i \lambda_i^{I*}\right)$ represents the Foreign labor content. A "dollar" amount of final import contains $(1 - \beta_i)$ "dollars" the Foreign downstream labor and $\beta_i \lambda_i^{I*}$ "dollars" of Foreign upstream labor. The domestic (Home) content is $\beta_i \left(1 - \lambda_i^{I*}\right)$. The intuition here is that Home wants to encourage Foreign to import more intermediates,[15] and in order to do so more final imports are allowed to reach Home consumers. Observe that a "dollar" spent by Home on importing final goods from Foreign increases Foreign final sales by one "dollar", intermediate spending by β_i "dollars" and Home exports by $\beta_i \left(1 - \lambda_i^{I*}\right)$ "dollars", of which $\frac{1}{tx_i^I} \beta_i \left(1 - \lambda_i^{I*}\right)$ is received by Home producers and $\frac{tx_i^I - 1}{tx_i^I} \beta_i \left(1 - \lambda_i^{I*}\right)$ by the Home government. In a sense, Home compensates Foreign consumers for the export taxes they effectively pay. This logic appears more straightforward under an alternative scaling of nominal variables. If instead of setting $W = W^* = 1$ we put $\widetilde{tm^I} = 1$ and $\frac{\widetilde{W^*}}{\widetilde{W}} = tm^I$, we have $\widetilde{tx_i^I} = tx_i^I tm^I$ and (3.20) becomes $\widetilde{tm_i^F} = 1 - \beta_i \left(1 - \lambda_i^{I*}\right) \frac{\widetilde{tx_i^I} - 1}{\widetilde{tx_i^I}}$. For positive intermediate export tax rates, $\widetilde{tx_i^I} > 1$, Home subsidizes Foreign final imports by returning Foreign consumers the revenue generated from intermediate exports.

3.2 Export taxes

We may relate the problem of setting export taxes to a more familiar price-setting problem of a multiproduct monopolist. Observe that the terms of (3.14) involving Px_i^I and Px_i^F are

$$\widetilde{\Lambda} = -\overline{\mu}^* \sum\nolimits_i \left(l_i^{F*} + l_i^{I*}\right) - \sum\nolimits_i \mu_i^F \tau_i^F c_{im}^* - \sum\nolimits_i \mu_i^I \tau_i^I x_{im}^*. \tag{3.21}$$

Appendix A.2 proves Proposition 3.3 stating that Foreign employment l_i^{F*} and l_i^{I*} in (3.21) can be expressed through Home export revenue, and therefore $\widetilde{\Lambda}$ resembles a

[15] Stronger import demand for Home intermediates is beneficial for two reasons. First, it translates into higher Foreign demand for Home labor, which is a general equilibrium terms-of-trade channel. (One unit of Home labor can be effectively exchanged for more units of Foreign labor.) Second, it increases Home opportunities to raise export tax revenue.

monopolist profit function.

Proposition 3.3. *The equation (3.21) can be expressed as*

$$\widetilde{\Lambda} = \sum\nolimits_i \left(\overline{\mu}^* Px_i^F - \mu_i^F \tau_i^F\right) c_{im}^* + \sum\nolimits_i \left(\overline{\mu}^* Px_i^I - \mu_i^I \tau_i^I\right) x_{im}^* - \sum\nolimits_i \overline{\mu}^* MC_i^{F*} \tau_i^F c_{im} + const, \tag{3.22}$$

where the final constant term does not depend on the export prices.

The first two terms in (3.22) are analogous to profit of a multiproduct monopolist. For final goods, for example, selling one unit of product i to Foreign generates $\overline{\mu}^* Px_i^F$ in terms of revenue while the costs of providing it are $\mu_i^F \tau_i^F$. The third term, however, is unique to our international production sharing setup. The Home government must take into account that intermediate export prices Px_i^I affect Foreign final marginal costs MC_i^{F*} and thus total labor costs of providing Home final imports c_{im}. Final export prices Px_i^F do not affect Foreign production costs, which allows a relatively simple characterization.

Differentiating (3.22) with respect to Px_i^F gives a standard-looking monopolist first-order condition

$$\overline{\mu}^* c_{im}^* + \left(\overline{\mu}^* Px_i^F - \mu_i^F \tau_i^F\right) \frac{\partial c_{im}^*}{\partial Px_i^F} + \left(\overline{\mu}^* Px_i^I - \mu_i^I \tau_i^I\right) \frac{\partial x_{im}^*}{\partial Px_i^F} = 0. \tag{3.23}$$

The last term in (3.23) signifies the presence of production linkages. The Home government needs to take into account the positive cross-price elasticity, as higher final export prices Px_i^F induce consumer switching in Foreign toward domestic varieties, which boosts the demand for imported intermediates x_{im}^*. We can define the elasticity of final Foreign import demand as

$$\varepsilon_i^F = -\frac{\partial \log c_{im}^*}{\partial \log Px_i^F}. \tag{3.24}$$

Similar to the variable markups literature that works with nested CES preferences, for example Amiti, Itskhoki, and Konings (2014), this elasticity can be decomposed as $\varepsilon_i^F = \left(1 - \lambda_i^{F*}\right) \cdot 1 + \lambda_i^{F*} \rho_i^F$. The industry-level elasticity of substitution ρ_i^F is combined with the Cobb-Douglas elasticity of substitution 1 with the weights depending on $\left(1 - \lambda_i^{F*}\right)$ which is the share of Home product in the Foreign market.

Appendix A.2 proves the following

Proposition 3.4. *Optimal export taxes for final goods satisfy*

$$tx_i^F = \frac{\varepsilon_i^F}{\varepsilon_i^F - 1} \frac{1}{tm_i^F} = \frac{\varepsilon_i^F}{\varepsilon_i^F - 1} \frac{1}{tm^I - \beta_i \left(1 - \lambda_i^{I*}\right) \left(tm^I - \frac{1}{tx_i^I}\right)}. \tag{3.25}$$

Two factors determine variation in final export taxes across industries. First, a higher Home market share in the Foreign market and hence a lower import demand elasticity ε_i^F implies a higher markup $\frac{\varepsilon_i^F}{\varepsilon_i^F - 1}$. These market shares are higher for stronger Home comparative advantage in either upstream or downstream, or in the case of lower trade costs τ_i^F. Second, as already mentioned, there is a consumer expenditure switching effect. We have $Px_i^F \uparrow \to \lambda_i^{F*} \uparrow \to y_i^{F*} \uparrow \to x_{im}^* \uparrow$: the Foreign demand for intermediate imports is increasing in final export prices. The strength of this channel varies with $\beta_i \left(1 - \lambda_i^{I*}\right)$, the share of Home-produced inputs in Foreign final output, and the exact expression is such that tm_i^F is the single summary statistic. Therefore, the optimal levels of export taxes tx_i^F are discounted by the values of import tariffs tm_i^F. Conditional on the demand elasticity ε_i^F, lower tariffs for final goods correspond to higher final export taxes. Such lower tariffs can arise, for example, for industries with larger intermediate shares β_i, which is a source of variation generally independent of ε_i^F.

Intermediate export prices Px_i^I are more complicated because of the impact on Foreign final marginal costs. [to be completed]

3.3 Comparative Statics

What is the role of comparative advantage for optimal trade policy in the presence of intermediate goods? To shed some light on this question, I perform the following numerical exercise. All parameters of the model other than cost shifters are set common across industries. The relative upstream and downstream cost shifters $\frac{Z_i^{I*}}{Z_i^I}$ and $\frac{Z_i^{F*}}{Z_i^F}$ are set on a rectangular grid. Therefore, for a given level of upstream (Home) cost advantage $\frac{Z_i^{I*}}{Z_i^I}$, we can look at variation of optimal trade taxes in response to variation in downstream cost advantage $\frac{Z_i^{F*}}{Z_i^F}$. (Not only the ratio $\frac{Z_i^{I*}}{Z_i^I}$ but also both variables are kept constant.) Similarly, we can look at the role of $\frac{Z_i^{I*}}{Z_i^I}$ keeping $\frac{Z_i^{F*}}{Z_i^F}$ fixed. Figure 1 presents the results which are typical for various levels of model parameters.

Similar to Costinot et al. (2015), export taxes increase monotonically in the level of (Home) comparative advantage. In our two-stage production setup, that applies to both final and intermediate export taxes, on the one hand, and, on the other hand,

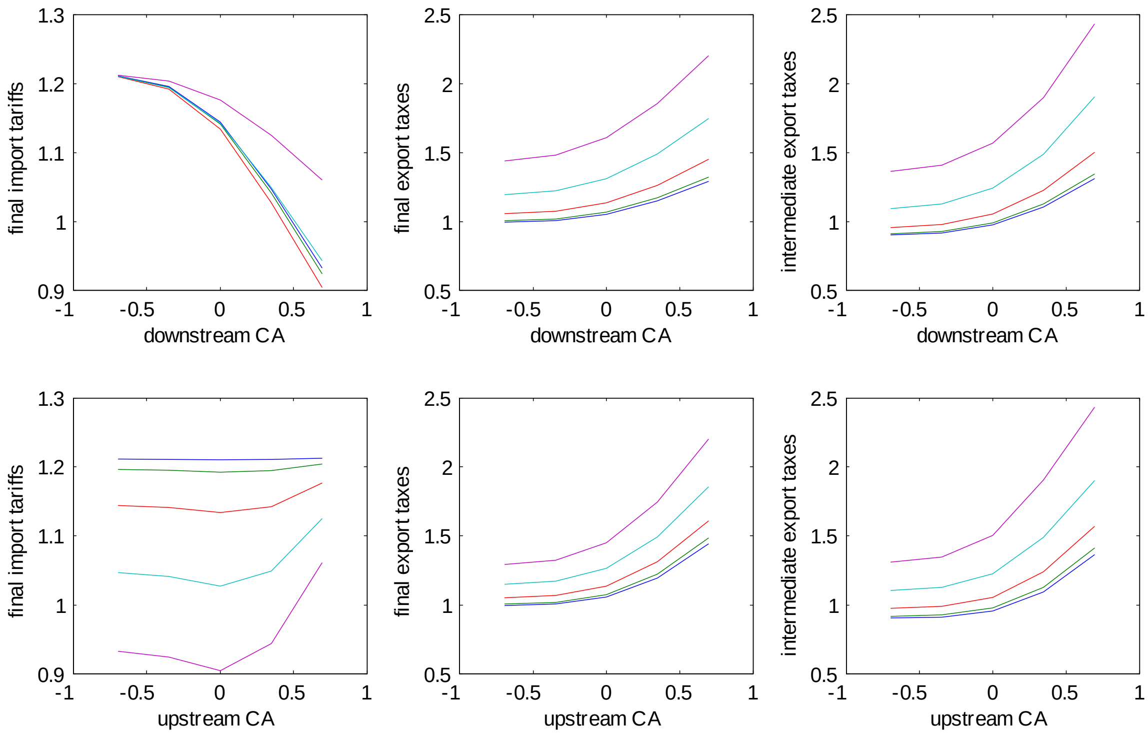

Figure 1: Variation in policy instruments in response to variation in downstream and upstream comparative advantage.

both upstream and downstream comparative advantage. Intuitively, stronger comparative advantage corresponds to larger market power and higher monopoly markups.

Final import tariffs are declining in downstream comparative advantage. From $tm_i^F = tm^I - \beta_i \left(1 - \lambda_i^{I*}\right) \left(tm^I - \frac{1}{tx_i^I}\right)$, $\frac{Z_i^{F*}}{Z_i^F}$ only affects tm_i^F via intermediate export taxes tx_i^I, both directly and indirectly through λ_i^{I*}. It appears that the negative direct effect always dominates the positive indirect effect, so final import tariffs decline in downstream comparative advantage. At the same time, final import tariffs are generally non-monotone in upstream comparative advantage. We have $\frac{Z_i^{I*}}{Z_i^I} \uparrow \rightarrow \lambda_i^{I*} \downarrow, tx_i^I \uparrow$, and the overall effect is ambiguous.

4 Conclusion

This paper applies the primal approach to optimal trade policy in a two-stage production environment. It finds that intermediate import tariffs are uniform while final import tariffs decline with domestic labor content. Final export taxes are determined by monopoly markups and discounted by the values of final import tariffs, due to the consumer expenditure switching effect. In general, stronger international production sharing reduces the incentives to impose import protection but increases the incentives to tax exports.

Considering more general input-output structures, the formula for import tariffs (3.20) generalizes easily, but export taxes become increasingly difficult to characterize. Unfortunately, this does not mean that we may abstract from export taxes, because that would break the results for import tariffs.

A Appendix

A.1 Foreign Equilibrium

Part I

First, upstream and downstream marginal costs are

$$MC_i^{I*} = Z_i^{I*} W^* \text{ and} \tag{A.1}$$

$$MC_i^{F*} = Z_i^{F*} \left(W^*\right)^{1-\beta_i} \left(P_{Mi}^*\right)^{\beta_i}, \tag{A.2}$$

where the price index for material input is

$$P^*_{Mi} = \left(\left(MC_i^{I*}\right)^{1-\rho_i^I} + \left(tx_i^I \tau_i^I MC_i^I\right)^{1-\rho_i^I}\right)^{\frac{1}{1-\rho_i^I}}. \tag{A.3}$$

From (A.3) we observe that intermediate export taxes imposed by Home reduce, tx_i^I, reduce downstream productivity in Foreign.

The Foreign final domestic share is

$$\lambda_i^{F*} = \frac{\left(MC_i^{F*}\right)^{1-\rho_i^F}}{\left(MC_i^{F*}\right)^{1-\rho_i^F} + \left(tx_i^F \tau_i^F MC_i^F\right)^{1-\rho_i^F}}. \tag{A.4}$$

and demand quantities are

$$c^*_{id} = \frac{\alpha_i \lambda_i^{F*} W^* L^*}{MC_i^{F*}} \text{ and } c^*_{im} = \frac{\alpha_i \left(1 - \lambda_i^{F*}\right) W^* L^*}{tx_i^F \tau_i^F MC_i^F}. \tag{A.5}$$

The revenue of final producers, y_i^{F*}, comes from serving domestic demand c^*_{id} and Home import demand c_{im} adjusted by trade costs τ_i^F:

$$y_i^{F*} = MC_i^{F*} \cdot \left(c^*_{id} + \tau_i^F c_{im}\right). \tag{A.6}$$

Final producers in industry i spend $(1-\beta_i)\, y_i^{F*}$ on labor and employ

$$l_i^{F*} = \frac{(1-\beta_i)\, y_i^{F*}}{W^*}. \tag{A.7}$$

Domestic shares for intermediate expenditure:

$$\lambda_i^{I*} = \frac{\left(MC_i^{I*}\right)^{1-\rho_i^I}}{\left(MC_i^{I*}\right)^{1-\rho_i^I} + \left(tx_i^I \tau_i^I MC_i^I\right)^{1-\rho_i^I}}. \tag{A.8}$$

Quantities demanded by intermediate producers are

$$x^*_{id} = \frac{\lambda_i^{I*} \beta_i y_i^{F*}}{MC_i^{I*}} \text{ and } x^*_{im} = \frac{\left(1 - \lambda_i^{I*}\right) \beta_i y_i^{F*}}{tx_i^I \tau_i^I MC_i^I}. \tag{A.9}$$

Revenue of intermediate producers:

$$y_i^{I*} = MC_i^{I*} \cdot \left(x^*_{id} + \tau_i^I x_{im}\right). \tag{A.10}$$

Employment in producing intermediates:

$$l_i^{I*} = \frac{y_i^{I*}}{W^*}. \tag{A.11}$$

Labor market clearing:

$$\sum_i \left(l_i^{I*} + l_i^{F*}\right) = L^*. \tag{A.12}$$

Part II

The Foreign equilibrium relationships (3.2)-(3.7) are similar to (A.1)-(A.11), except for the following modifications.

(i) A normalization $W^* = 1$ is imposed.

(ii) We have Px_i^I instead of $tx_i^I \tau_i^I MC_i^I$ and Px_i^F instead of $tx_i^F \tau_i^F MC_i^F$, the planner directly sets export prices.

(iii) The final marginal costs MC_i^{F*} are expressed as $Z_i^{F*}\left(Z_i^{I*}\right)^{\beta_i}\left(\lambda_i^{I*}\right)^{\frac{\beta_i}{\rho_i^I - 1}}$. See derivation of (2.22) in the main text of the paper.

A.2 Proof of Proposition 3.3

We use that a country's production is equal to domestic absorption plus net export. For final goods this accounting identity is

$$y_i^{F*} = \alpha_i L^* + MC_i^{F*} \tau_i^F c_{im} - Px_i^F c_{im}^*$$

and for intermediate goods it is

$$y_i^{I*} = \beta_i y_i^{F*} + MC_i^{I*} \tau_i^I x_{im} - Px_i^I x_{im}^*.$$

Therefore,

$$l_i^{F*} + l_i^{I^*} = (1-\beta_i)\, y_i^{F*} + y_i^{I*} = (1-\beta_i)\, y_i^{F*} + \beta_i y_i^{F*} + MC_i^{I*}\tau_i^I x_{im} - Px_i^I x_{im}^* =$$

$$y_i^{F*} + MC_i^{I*}\tau_i^I x_{im} - Px_i^I x_{im}^* = \alpha_i L^* + MC_i^{F*}\tau_i^F c_{im} - Px_i^F c_{im}^* + MC_i^{I*}\tau_i^I x_{im} - Px_i^I x_{im}^*.$$

Among the terms of the last expression, $\alpha_i L^*$ and $MC_i^{I*}\tau_i^I x_{im}$ are not affected by export prices. By substituting $l_i^{F*} + l_i^{I^*}$ in (3.21) with that expression we get (3.22). ■

A.3 Proof of Proposition 3.4

We start with the first-order condition

$$\overline{\mu}^* c_{im}^* + \left(\overline{\mu}^* Px_i^F - \mu_i^F \tau_i^F\right) \frac{\partial c_{im}^*}{\partial Px_i^F} + \left(\overline{\mu}^* Px_i^I - \mu_i^I \tau_i^I\right) \frac{\partial x_{im}^*}{\partial Px_i^F} = 0.$$

Dividing by $\overline{\mu}^* c_{im}^*$ yields

$$1+\left(1-\frac{\mu_i^F\tau_i^F}{\overline{\mu}^* Px_i^F}\right)\frac{\partial c_{im}^*}{\partial Px_i^F}\frac{Px_i^F}{c_{im}^*}+\left(1-\frac{\mu_i^I\tau_i^I}{\overline{\mu}^* Px_i^I}\right)\frac{\partial x_{im}^*}{\partial Px_i^F}\frac{Px_i^I}{c_{im}^*}=0.$$

As $\mu_i^F=\overline{\mu}MC_i^F$, $\mu_i^I=\overline{\mu}MC_i^I$ and $\frac{\overline{\mu}^*}{\overline{\mu}}=tm^I$, the ratio $\frac{\mu_i^F\tau_i^F}{\overline{\mu}^* Px_i^F}$ is equal to $\frac{1}{tm^I tx_i^F}$ and the ratio $\frac{\mu_i^I\tau_i^I}{\overline{\mu}^* Px_i^I}$ equals to $\frac{1}{tm^I tx_i^I}$. Our equation becomes

$$1-\left(1-\frac{1}{tm^I tx_i^F}\right)\varepsilon_i^F+\left(1-\frac{1}{tm^I tx_i^I}\right)\frac{\partial x_{im}^*}{\partial Px_i^F}\frac{Px_i^I}{c_{im}^*}=0. \tag{A.13}$$

We next use that $Px_i^I x_{im}^*=\left(1-\lambda_i^{I*}\right)\beta_i y_i^{F*}=\left(1-\lambda_i^{I*}\right)\beta_i\left(\alpha_i L^*+MC_i^{F*}\tau_i^F c_{im}-Px_i^F c_{im}^*\right)$, so $\frac{\partial x_{im}^*}{\partial Px_i^F}=-\frac{1}{Px_i^I}\left(1-\lambda_i^{I*}\right)\beta_i\frac{\partial\left(Px_i^F c_{im}^*\right)}{\partial Px_i^F}=-\left(1-\lambda_i^{I*}\right)\beta_i\left(1-\varepsilon_i^F\right)\frac{c_{im}^*}{Px_i^I}$. Once we plug in this expression into (A.13), we have

$$1-\left(1-\frac{1}{tm^I tx_i^F}\right)\varepsilon_i^F+\left(1-\frac{1}{tm^I tx_i^I}\right)\left(1-\lambda_i^{I*}\right)\beta_i\left(\varepsilon_i^F-1\right)=0, \text{ from which}$$

$$tx_i^F=\frac{\varepsilon_i^F}{\varepsilon_i^F-1}\frac{1}{tm^I-\left(1-\lambda_i^{I*}\right)\beta_i\left(tm^I-\frac{1}{tx_i^I}\right)}.$$

References

[1] Baldwin, Richard. 2012. "WTO 2.0: Global governance of supply-chain trade." CEPR Policy Insight 64.

[2] Blanchard, Emily J., Chad P. Bown, and Robert C. Johnson. 2016. "Global supply chains and trade policy." No. w21883. National Bureau of Economic Research.

[3] Costinot, Arnaud, Dave Donaldson, Jonathan Vogel, and Iván Werning. 2015. Comparative Advantage and Optimal Trade Policy. The Quarterly Journal of Economics, 130, 659-702.

[4] Amiti, Mary, Oleg Itskhoki, and Jozef Konings. 2014. "Importers, exporters, and exchange rate disconnect." The American Economic Review 104.7 1942-1978.

[5] Opp, Marcus M. 2010. "Tariff wars in the Ricardian Model with a continuum of goods." Journal of International Economics 80.2: 212-225.

[6] Yi, K. 2003. "Can Vertical Specialization Explain the Growth of World Trade?" Journal of Political Economy.

[7] Yi, K., 2010. Can Multi-Stage Production Explain the Home Bias in Trade? American Economic Review 100, pp. 364-393.