Comparative advantage in routine production *

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Abstract

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1 Introduction

2 Stylized facts

3 The model

3.1 Autarky

The two countries are denoted $i \in \{1,2\}$. Factor endowments of capital $\bar{K}$ and labor $\bar{L}$ are common to the two countries. The two final goods are denoted $g \in \{1,2\}$. The production function for good g in country i is:

$$Y_{ig} = z_g (L^a_{ig})^{1-\beta_g} M_{ig}^{\beta_g} \tag{1}$$

where z_g is the technology parameter in production of good g; L^a_{ig} is the quantity of non-routine labor used in production of good g, M_{ig} is the quantity of the routine input used in production of good g, and β_g is the factor share of the routine input in production of good g. Throughout this section, we consider good 1 to be relatively non-routine: $\beta_1 < \beta_2$.

The Cobb-Douglas production function for each of the two final goods contains an inner component that describes how routine labor L^m_{ig} and capital K_{ig} are combined to produce the routine input M_{ig}:

$$M_{ig} = A_i \left[\alpha_i {K_{ig}}^{\mu_i} + (1-\alpha_i)(L^m_{ig})^{\mu_i} \right]^{1/\mu_i} \tag{2}$$

where A_i and α_i are (respectively) the efficiency and distribution parameters of the CES production function, and $\mu_i = (\sigma_i - 1)/\sigma_i$ captures the extent of capital-routine labor substitutability. These three parameters may be country-specific. Throughout this section, we consider country 1 to have higher substitutability between capital and routine labor: $\mu_1 > \mu_2$. Further, we assume that in both countries capital and routine labor are more substitutable than non-routine labor and the routine input: $0 < \mu_i < 1$.

The full expression of the production function is:

$$Y_{ig} = z_g (L^a_{ig})^{1-\beta_g} \left\{ A_i \left[\alpha_i {K_{ig}}^{\mu_i} + (1-\alpha_i)(L^m_{ig})^{\mu_i} \right]^{1/\mu_i} \right\}^{\beta_g} \tag{3}$$

We denote $M_i = \sum_g M_{ig}$ the total quantity of the routine input and $L^m_i = \sum_g L^m_{ig}$ the total quantity of labor allocated to its production. We denote $L^a_i = \bar{L} - L^m_i = \sum_g L^a_{ig}$ the total quantity of labor allocated to non routine tasks. The outer production function (1) replicates the canonical

2×2 Heckscher-Ohlin model with two goods and two factors. Hence, it is sufficient to establish that one of the two countries becomes relatively non-routine labor abundant to prove that under autarky this country produces relatively more output in the sector that uses non-routine labor more intensively: $L_i^a/M_i > L_{i'}^a/M_{i'} \Leftrightarrow Y_{i1}/Y_{i2} > Y_{i'1}/Y_{i'2}$. The objective of this section is to establish the conditions under which the high (low) -μ country becomes relatively non-routine labor abundant.

3.1.1 Production of the routine input (M_i)

We denote w_i the wage and r_i the cost of capital and posit that for any given relative wage w_i/r_i, routine labor and capital are combined in the same way in production of the two final goods.[1] Further, we note that capital can only be used to produce the routine input. The production function for the routine input M_i can thus be written:

$$M_i = A_i\,[\alpha_i\bar{K}^{\mu_i} + (1-\alpha_i)(L_i^m)^{\mu_i}]^{1/\mu_i} \tag{4}$$

We use (4) to solve for the use of routine labor as a function of routine input production. Whenever $M_i \le A_i\alpha_i^{\frac{1}{\mu_i}}\bar{K}$, $L_i^m = 0$. Whenever $M_i > A_i\alpha_i^{\frac{1}{\mu_i}}\bar{K}$:

$$L_i^m(M_i) = [1-\alpha_i]^{-\frac{1}{\mu_i}}\left[\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i\bar{K}^{\mu_i}\right]^{1/\mu_i} \tag{5}$$

We denote by C_i^m the cost function for the routine input. For $M_i \in \left(0, A_i\alpha_i^{\frac{1}{\mu_i}}\bar{K}\right]$ the cost function is constant and equal to $r_i\bar{K}$. For $M_i > A_i\alpha_i^{\frac{1}{\mu_i}}\bar{K}$ the cost function is:

$$C_i^m(M_i;w_i,r_i) = w_iL_i^m + r_i\bar{K} = w_i\,[1-\alpha_i]^{-\frac{1}{\mu_i}}\left[\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i\bar{K}^{\mu_i}\right]^{1/\mu_i} + r_i\bar{K} \tag{6}$$

Consequently, the marginal cost function is constant and equal to 0 for $M_i \le A_i\alpha_i^{\frac{1}{\mu_i}}\bar{K}$. When M_i exceeds $A_i\alpha_i^{\frac{1}{\mu_i}}\bar{K}$, the marginal cost function is:

$$\{C_i^m(M_i;w_i,r_i)\}' = MC_i^m(M_i;\cdot) = w_i\,[1-\alpha_i]^{-\frac{1}{\mu_i}}A_i^{-\mu_i}M_i^{\mu_i-1}\left[\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i\bar{K}^{\mu_i}\right]^{\frac{1-\mu_i}{\mu_i}} \tag{7}$$

The average cost function for $M_i \in \left(0, A_i\alpha_i^{1/\mu_i}\bar{K}\right]$ is given by $r_i\bar{K}/M_i$. It is decreasing from $+\infty$ to $r_iA_i^{-1}\alpha_i^{-1/\mu_i}$. One would always produce at least $M_i = A_i\alpha_i^{1/\mu_i}\bar{K}$ since the marginal cost is 0 in this range. We therefore focus on the segment that verifies $M_i > A_i\alpha_i^{1/\mu_i}\bar{K}$:

[1] App.A.1 validates this assumption by establishing the uniqueness of the solution.

$$\frac{C_i^m(M_i;w_i,r_i)}{M_i} = AC_i^m(M_i;\cdot) \;=\; \frac{w_i}{M_i}[1-\alpha_i]^{-\frac{1}{\mu_i}}\left[\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i\bar{K}^{\mu_i}\right]^{1/\mu_i} + \frac{r_i\bar{K}}{M_i} \tag{8}$$

When the quantity produced approaches infinity, the average cost function approaches the marginal cost function:

$$\lim_{M_i\to\infty} AC_i^m(\cdot) = \lim_{M_i\to\infty}\left\{\frac{w_i}{M_i}[1-\alpha_i]^{-\frac{1}{\mu_i}}\left[\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i\bar{K}^{\mu_i}\right]^{1/\mu_i}\right\} =$$
$$\lim_{M_i\to\infty}\left\{w_i[1-\alpha_i]^{-\frac{1}{\mu_i}}A_i^{-\mu_i}M_i^{\mu_i-1}\left[\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i\bar{K}^{\mu_i}\right]^{(1-\mu_i)/\mu_i}\right\} = w_i[1-\alpha_i]^{-\frac{1}{\mu_i}}A_i^{-1} \tag{9}$$

We check that the average cost function attains a minimum for a unique and finite value of M_i. The derivative of the average cost function is:

$$\frac{dAC_i^m(M_i;w_i,r_i)}{dM_i} \;=\; \frac{w_i(1-\alpha_i)^{-\frac{1}{\mu_i}}\alpha_i\bar{K}^{\mu_i}\left[\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i\bar{K}^{\mu_i}\right]^{\frac{1-\mu_i}{\mu_i}} - r_i\bar{K}}{M_i^2} \tag{10}$$

The derivative is non-negative when:

$$w_i(1-\alpha_i)^{-\frac{1}{\mu_i}}\alpha_i\bar{K}^{\mu_i}\left[\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i\bar{K}^{\mu_i}\right]^{\frac{(1-\mu_i)}{\mu_i}} - r_i\bar{K} \geq 0 \tag{11}$$

Rearranging and simplifying gives:

$$\left[\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i\bar{K}^{\mu_i}\right] \geq \left[\frac{w_i}{r_i}\right]^{\frac{-\mu_i}{1-\mu_i}}\left[\frac{1-\alpha}{\alpha}\right]^{\frac{1}{1-\mu_i}}\alpha_i\bar{K}^{\mu_i} \tag{12}$$

We solve for M_i to show that the average cost function is increasing whenever:

$$M_i \geq A_i\alpha_i^{\frac{1}{\mu_i}}\bar{K}\left\{1+\left[\frac{w_i}{r_i}\right]^{\frac{-\mu_i}{1-\mu_i}}\left[\frac{1-\alpha_i}{\alpha_i}\right]^{\frac{1}{1-\mu_i}}\right\}^{\frac{1}{\mu_i}} \tag{13}$$

The term in the curly brackets is strictly bigger than 1. Consequently, the average cost function is decreasing between $A_i\alpha_i^{\frac{1}{\mu_i}}\bar{K}$ and $A_i\alpha_i^{\frac{1}{\mu_i}}\bar{K}\left\{1+[w_i/r_i]^{\frac{-\mu_i}{1-\mu_i}}[(1-\alpha_i)/\alpha_i]^{\frac{1}{1-\mu_i}}\right\}^{1/\mu_i}$ and increasing thereafter. Thus, the average cost of producing the routine input is minimized at:

$$M_i^* \;=\; A_i\alpha_i^{\frac{1}{\mu_i}}\bar{K}\left\{1+\left[\frac{w_i}{r_i}\right]^{\frac{-\mu_i}{1-\mu_i}}\left[\frac{1-\alpha_i}{\alpha_i}\right]^{\frac{1}{1-\mu_i}}\right\}^{\frac{1}{\mu_i}} \tag{14}$$

This result indicates that for any set of finite factor prices the optimal choice of routine input production M_i^* is strictly bigger than $A_i\alpha_i^{\frac{1}{\mu_i}}\bar{K}$. It follows that some labor will be allocated to the production of the routine input in each country as long as μ_i is bounded away from 1.

Plugging (14) into (5) delivers the optimal quantity of routine labor:[2]

$$L_i^{m*} = \left(\frac{w_i}{r_i}\right)^{-\frac{1}{1-\mu_i}} \left(\frac{1-\alpha_i}{\alpha_i}\right)^{\frac{1}{1-\mu_i}} \bar{K} \tag{15}$$

Using labor market clearing together with the condition that ensures optimal routine input production (14), we obtain the optimal ratio of non routine labor to the routine input for country i derived from the solution of the inner problem:

$$\frac{L_i^a(M_i{}^*)}{M_i^*} = \frac{\bar{L} - L_i^m(M_i^*)}{M_i^*} = \frac{\bar{L} - \left[\frac{w_i/(1-\alpha_i)}{r_i/\alpha_i}\right]^{-\frac{1}{1-\mu_i}} \bar{K}}{A_i \alpha_i^{\frac{1}{\mu_i}} \bar{K} \left\{1 + \frac{w_i}{r_i}\left[\frac{w_i/(1-\alpha_i)}{r_i/\alpha_i}\right]^{-\frac{1}{1-\mu_i}}\right\}^{\frac{1}{\mu_i}}} \tag{16}$$

The solution to the inner problem also delivers the price of the routine input. Plugging the optimal choice of labor allocation to routine tasks (15) in the cost function (6) gives:

$$C_i^m(M_i^*; w_i, r_i) = r_i \bar{K} \left[1 + \left(\frac{w_i}{r_i}\right)\left(\frac{w_i/(1-\alpha_i)}{r_i/\alpha_i}\right)^{-\frac{1}{1-\mu_i}}\right] \tag{17}$$

Consequently, the price of the routine input is:

$$P_i^m(M_i^*; \cdot) = \frac{C_i^m(M_i^*)}{M_i^*} = \frac{r_i \alpha_i^{-\frac{1}{\mu_i}}}{A_i} \left[1 + \frac{w_i}{r_i}\left(\frac{w_i/(1-\alpha_i)}{r_i/\alpha_i}\right)^{-\frac{1}{1-\mu_i}}\right]^{\frac{\mu_i - 1}{\mu_i}} \tag{18}$$

We rearrange (18) to demonstrate that the price of the routine input mimicks the CES price index that would be obtained if both production factors were freely chosen:

$$\begin{aligned} P_i^m(M_i^*; \cdot) = A_i^{-1} \alpha_i^{\frac{\mu_i - 1}{\mu_i}} \frac{r_i}{\alpha_i} \left[\left(\frac{r_i/\alpha_i}{r_i/\alpha_i}\right)^{\frac{\mu_i}{\mu_i - 1}} + \left(\frac{1-\alpha_i}{\alpha_i}\right)\left(\frac{w_i/(1-\alpha_i)}{r_i/\alpha_i}\right)^{\frac{\mu_i}{\mu_i - 1}}\right]^{\frac{\mu_i - 1}{\mu_i}} &= \\ A_i^{-1} \alpha_i^{\frac{\mu_i - 1}{\mu_i}} \left[\left(\frac{r_i}{\alpha_i}\right)^{\frac{\mu_i}{\mu_i - 1}} + \left(\frac{1-\alpha_i}{\alpha_i}\right)\left(\frac{w_i}{1-\alpha_i}\right)^{\frac{\mu_i}{\mu_i - 1}}\right]^{\frac{\mu_i - 1}{\mu_i}} &= \\ A_i^{-1} \left[\alpha_i \left(\frac{r_i}{\alpha_i}\right)^{\frac{\mu_i}{\mu_i - 1}} + (1-\alpha_i)\left(\frac{w_i}{1-\alpha_i}\right)^{\frac{\mu_i}{\mu_i - 1}}\right]^{\frac{\mu_i - 1}{\mu_i}} = A_i^{-1}\left[\alpha_i^{\sigma_i} r_i{}^{1-\sigma_i} + (1-\alpha_i)^{\sigma_i} w_i{}^{1-\sigma_i}\right]^{\frac{1}{1-\sigma_i}} & \end{aligned} \tag{19}$$

Denote the effective relative cost of labor by $\varpi_i = [w_i/(1-\alpha_i)]/[r_i/\alpha_i]$. From (15), the elasticity of substitution between capital and routine labor at the cost-minimizing choice of routine input production is $\sigma_i = d\ln[\bar{K}/L_i^{m*}]/d\ln(\varpi) = (1-\mu_i)^{-1}$. If the effective cost of labor were common to the two countries, the production of the routine input would be relatively

[2] This quantity corresponds to the relative factor demand that would be chosen to produce M_i^* if both capital and routine labor were freely chosen.

labor-intensive in the high-μ country whenever the effective cost of labor is relatively low ($\varpi < 1$). Further, from the general mean property of the CES production function we know that the quantity of the routine output M_i is strictly increasing in μ whenever the two countries allocate the same combination of inputs $\{K, L^m\}$ to its production and $K \neq L^m$ (Klump and De La Grandville (2012)).[3] Whenever labor is relatively cheap $\varpi < 1$, we know from (15) that the high-μ country allocates relatively more labor to routine tasks $L_1^{m*} > L_2^{m*}$ and from the general mean property that $M_1(L_2^{m*}) > M_2(L_2^{m*})$. It follows that the high-μ country must be relatively routine abundant: $L_1^a/M_1 < L_2^a/M_2$.

Whenever labor is relatively expensive $\varpi > 1$, we know from (15) that the high-μ country allocates relatively less labor to routine tasks $L_1^{m*} < L_2^{m*}$ whereby $L_1^{a*} > L_2^{a*}$. However, this is insufficient to prove that the high-μ country is relatively non routine abundant because from the general mean property we only know that $M_1(L_1^{m*}) > M_2(L_1^{m*})$. If we compute the derivative of M_i^* with respect to μ while keeping ϖ fixed, we get:

$$\frac{\partial \ln M_i^*}{\partial \mu} = \frac{d}{d\mu}\left\{\frac{1}{\mu}\ln\left[\alpha_i\left(1+\frac{w_i}{r_i}\varpi^{-\frac{1}{1-\mu}}\right)\right]\right\} = -\mu^{-2}(1-\mu)^{-2}\left(1+\frac{w_i}{r_i}\varpi_i^{-\frac{1}{1-\mu}}\right)^{-1}$$
$$\left\{(1-\mu)^2\left[1+\frac{w_i}{r_i}\varpi^{-\frac{1}{1-\mu}}\right]\ln\left[\alpha_i\left(1+\frac{w_i}{r_i}\varpi^{-\frac{1}{1-\mu}}\right)\right]+\mu\frac{w_i}{r_i}\varpi^{-\frac{1}{1-\mu}}\ln\varpi\right\}$$

We can show that $\left[\alpha_i\left(1+\frac{w_i}{r_i}\varpi^{-\frac{1}{1-\mu}}\right)\right] < 1$ iff $\varpi_i > 1$. Consequently, the first term in the curly brackets is strictly negative while the second term is strictly positive whenever $\varpi > 1$. It is sufficient to prove that the expression in curly brackets is non-negative to establish that the high-μ country is non-routine labor abundant: $L_1^a/M_1 > L_2^a/M_2$ whenever $\varpi > 1$ (given $M_1(L_1^{m*}) \leq M_2(L_2^{m*})$. However, the sign of the expression in curly brackets is ambiguous.[4]

It is important to realize that studying the partial derivative of routine input production with respect to μ does not inform us on the pattern of specialization because, as we show in the next section, the equilibrium wage is itself a function of μ.[5] Further, the pattern of specialization is established through the ratio $(L_1^a/M_1)^*/(L_2^a/M_2)^*$ rather than through the ratio M_1^*/M_2^*. Specifically, to prove that the high-μ country is non-routine abundant whenever labor is scarce, we need to establish that $L_1^{a*}/L_2^{a*} > M_1^*/M_2^*$.

The inner problem does not suffice to pin down the factor price ratio. Hence, we consider

[3] In the very special case of $K = L^m$ ($\varpi = 1$) the two countries obtain the same amount of the routine input.

[4] We can prove that the expression in curly brackets is negative whenever $\varpi \leq (1-\alpha_i)^{\frac{-(1-\mu)^2}{\mu(2-\mu)}}$. Further, the expression is likely to be negative even if the above inequality does not hold.

[5] The equilibrium factor prices must verify the first order conditions for the inner CES and the outer Cobb-Douglas production function in each country. The factor price ratio is country-specific whenever $\varpi_i \neq 1$.

the outer problem to obtain the second expression of the optimal ratio of non routine labor to the routine input.

3.1.2 Production of final output

The outer problem is standard Heckscher-Ohlin. Costs are minimized in production of final good g by choosing M_{ig} and L^a_{ig} subject to the technological constraint $Y_{ig} \leq z_g \left(L^a_{ig}\right)^{1-\beta_g} {M_{ig}}^{\beta_g}$ taking factor prices P^m_i and w_i as given. The solution to this problem delivers relative factor demand:

$$\frac{L^a_{ig}}{M_{ig}} = \frac{1-\beta_g}{\beta_g}\frac{P^m_i}{w_i} \tag{20}$$

Denote Q_{ig} the consumption and P_{ig} the price of each final good. Goods' market clearing $Q_{ig} = Y_{ig}$ delivers $Q_{ig} = z_g \left(L^a_{ig}\right)^{1-\beta_g} {M_{ig}}^{\beta_g}$. Using (20) to substitute for M_{ig} defines the consumption of each good as a function of non-routine labor used in production:

$$Q_{ig} = z_g L^a_{ig} \left(\frac{\beta_g}{1-\beta_g}\right)^{\beta_g} \left(\frac{w_i}{P^m_i}\right)^{\beta_g} \tag{21}$$

Using the zero profit condition $P_{ig} z_g \left(L^a_{ig}\right)^{1-\beta_g} {M_{ig}}^{\beta_g} = w_i L^a_{ig} + P^m_i M_{ig}$ together with (20) to substitute for $P^m_i M_{ig}$ allows solving for the price of each final good:

$$P_{ig} = \frac{w_i^{1-\beta_g} (P^m_i)^{\beta_g}}{z_g (\beta_g)^{\beta_g} (1-\beta_g)^{(1-\beta_g)}} \tag{22}$$

We get the second expression of the ratio of non-routine labor to routine input in each country by considering the consumer problem. We take a standard Cobb-Douglas utility function for the two final goods: $U_i = \sum_g \theta_g \ln(Q_{ig})$. The budget constraint is $\sum_g P_{ig} Q_{ig} \leq r_i \bar{K} + w_i \bar{L}$. The solution to the consumer problem gives an expression of total expenditure on one good as a function of relative income shares of each good and expenditure on the other good:

$$P_{i2} Q_{i2} = \frac{\theta_2}{\theta_1} P_{i1} Q_{i1} \tag{23}$$

Plugging (21) and (22) into (23) gives non-routine labor allocation to one sector as a function of labor allocation to the other sector:

$$L^a_{i2} = \frac{\theta_2}{\theta_1}\frac{1-\beta_2}{1-\beta_1} L^a_{i1} \tag{24}$$

Using (24) together with $L^a_i = L^a_{i1} + L^a_{i2}$ allows expressing non-routine labor use in sector 1 as a function of total non-routine labor use:

$$L^a_{i1} = L^a_i \frac{\theta_1(1-\beta_1)}{\theta_1(1-\beta_1) + \theta_2(1-\beta_2)} \tag{25}$$

Using (20) to substitute for L_{ig}^{a} in (21) and plugging the resulting expression for Q_{ig} in (23) delivers the equivalent of (25) for the use of routine input in sector 1 as a function of total routine input use:

$$M_{i1} = M_i \frac{\theta_1 \beta_1}{\theta_1 \beta_1 + \theta_2 \beta_2} \tag{26}$$

Using (20) together with (25) and (26) gives an expression for the optimal relative use of non-routine labor and the routine input as a function of the factor price ratio:

$$\frac{L_i^{a*}}{M_i^*} = \frac{\sum_g \theta_g (1-\beta_g)}{\sum_g \theta_g \beta_g} \frac{P_i^m}{w_i} \tag{27}$$

We denote $c = \frac{\sum_g \theta_g (1-\beta_g)}{\sum_g \theta_g \beta_g}$ and replace the price of the routine input by its value in (18) to get:

$$\frac{L_i^{a*}}{M_i^*} = c \left[\frac{w_i}{r_i} A_i \alpha_i^{\frac{1}{\mu_i}} \right]^{-1} \left[1 + \left(\frac{w_i}{r_i} \right) \left(\frac{w_i/(1-\alpha_i)}{r_i/\alpha_i} \right)^{\frac{-1}{1-\mu_i}} \right]^{\frac{\mu_i - 1}{\mu_i}} \tag{28}$$

In the next section we pin down the equilibrium factor price ratio by combining the expression of optimal factor allocation in the production of the routine input with optimal factor allocation in the production of the two final goods. We thereafter evaluate the ratio $(L_1^{a*}/M_1^*)/(L_2^{a*}/M_2^*)$ as a function of μ.

3.1.3 Equilibrium factor price ratio

The solution to the inner and outer problems each deliver an expression for the relative use of non-routine labor and of the routine input in final good production as a function of the factor price ratio and of capital-routine labor substitutability. We solve for the equilibrium factor price ratio by equating (28) with (16):

$$\left(\frac{w_i}{r_i} \right)^{-1} c \left[1 + \left(\frac{w_i}{r_i} \right) \left(\frac{w_i/(1-\alpha_i)}{r_i/\alpha_i} \right)^{\frac{-1}{1-\mu_i}} \right]^{\frac{\mu_i - 1}{\mu_i}} = \frac{\left[\frac{\bar{L}}{\bar{K}} - \left(\frac{w_i/(1-\alpha_i)}{r_i/\alpha_i} \right)^{\frac{-1}{1-\mu_i}} \right]}{\left[1 + \left(\frac{w_i}{r_i} \right) \left(\frac{w_i/(1-\alpha_i)}{r_i/\alpha_i} \right)^{\frac{-1}{1-\mu_i}} \right]^{\frac{1}{\mu_i}}}$$

Rearranging and simplifying gives:

$$\left(\frac{w_i}{r_i} \right)^{-1} c \left[1 + \left(\frac{w_i}{r_i} \right) \left(\frac{w_i/(1-\alpha_i)}{r_i/\alpha_i} \right)^{\frac{-1}{1-\mu_i}} \right] = \left[\frac{\bar{L}}{\bar{K}} - \left(\frac{w_i/(1-\alpha_i)}{r_i/\alpha_i} \right)^{\frac{-1}{1-\mu_i}} \right]$$

$$\left(\frac{w_i}{r_i} \right)^{-1} c + c \left(\frac{w_i/(1-\alpha_i)}{r_i/\alpha_i} \right)^{\frac{-1}{1-\mu_i}} = \frac{\bar{L}}{\bar{K}} - \left(\frac{w_i/(1-\alpha_i)}{r_i/\alpha_i} \right)^{\frac{-1}{1-\mu_i}}$$

We obtain an implicit solution for the equilibrium factor price ratio $\omega_i^* = (w_i/r_i)^*$:

$$\omega_i^* \quad = \quad c\left[\frac{\bar{L}}{\bar{K}} - (1+c)\left[\frac{1-\alpha_i}{\alpha_i}\right]^{\frac{1}{1-\mu_i}} (\omega_i^*)^{\frac{-1}{1-\mu_i}}\right]^{-1} \tag{29}$$

To establish existence and uniqueness of the solution, we define $F_i(\cdot)$:

$$F_i\left(\omega_i^*;\mu_i,\frac{\bar{L}}{\bar{K}},c,\alpha_i,A_i\right) = (\omega_i^*)^{-1}c + (1+c)\left[(\omega_i^*)^{-\frac{1}{1-\mu_i}}\left(\frac{1-\alpha_i}{\alpha_i}\right)^{\frac{1}{1-\mu_i}}\right] - \frac{\bar{L}}{\bar{K}} = 0 \tag{30}$$

We eliminate negative exponents by factoring out $(\omega_i^*)^{-\frac{1}{1-\mu_i}}$ and use $\sigma_i = (1-\mu_i)^{-1}$ and $\sigma_i - 1 = \mu_i/(1-\mu_i)$ to show that the solution is the root of the polynomial of degree σ_i:

$$\begin{aligned} F_i\left(\omega_i;\mu_i,\frac{\bar{L}}{\bar{K}},c,\alpha_i,A_i\right) &= -\frac{\bar{L}}{\bar{K}}(\omega_i^*)^{\frac{1}{1-\mu_i}} + c\,(\omega_i^*)^{\frac{\mu_i}{1-\mu_i}} + (1+c)\left(\frac{1-\alpha_i}{\alpha_i}\right)^{\frac{1}{1-\mu_i}} = 0 \\ \Leftrightarrow\ & \frac{\bar{L}}{\bar{K}}(\omega_i^*)^{\sigma_i} - c\,(\omega_i)^{\sigma_i-1} - (1+c)\left(\frac{1-\alpha_i}{\alpha_i}\right)^{\sigma_i} = 0 \end{aligned} \tag{31}$$

The derivative with respect to ω_i^* is:

$$\frac{\partial F(\cdot)}{\partial \omega_i^*} \quad = \quad -\sigma_i(\omega_i^*)^{\sigma_i-1}\frac{\bar{L}}{\bar{K}} + c(\sigma_i-1)(\omega_i^*)^{\sigma_i-2} = -\sigma_i(\omega_i^*)^{\sigma_i-1}\left[\frac{\bar{L}}{\bar{K}} - c(\omega_i^*)^{-1}\right] - c\,(\omega_i^*)^{\sigma_i-2}$$

A sufficient condition for this derivative to be negative is to verify $\left[\bar{L}/\bar{K} - c(\omega_i^*)^{-1}\right] \geq 0$ or, equivalently, $\omega_i^* \geq c\bar{K}/\bar{L}$. By assumption, $\sigma_i \in (1,\infty)$. The function $F(\cdot)$ is monotonically decreasing in ω_i^*, it is positive for $\omega_i^* \to 0$ and negative for $\omega_i^* \to \infty$. We conclude that whenever $\omega_i^* \geq c\bar{K}/\bar{L}$, there exists a positive solution, and it gives rise to a finite real root ω_i^* that is the unique solution of this polynomial in each country.

The degree of the polynomial is country specific, and the solution to any polynomial in terms of its coefficients is degree-specific. Nevertheless, given the uniqueness of the solution, we can always express the solution of the polynomial in country 1 as a function of the solution in country 2: $\omega_1^* = \omega_2^*/\nu$.

3.1.4 Quantities and prices in autarky

We now characterize the pattern of production in autarky. From the consumer problem:

$$\frac{Q_{i1}}{Q_{i2}} = \rho\left[\frac{L_i^a}{M_i}\right]^{\beta_2-\beta_1} \tag{32}$$

where $\rho = (z_1\theta_1)/(z_2\theta_2)c^{\beta_1-\beta_2}(1-\beta_1)^{1-\beta_1}\beta_1^{\beta_1}(1-\beta_2)^{\beta_2-1}\beta_2^{-\beta_2}$.

Since ρ is invariant across countries and $\beta_2 > \beta_1$ by assumption, it is sufficient to show that one country is relatively non-routine labor abundant to prove that under autarky this country will have relatively high consumption of the good that uses non-routine labor more intensively. Equivalently, we can investigate in which country the relative price of the routine input $(P_i^m/w_i)^*$ is higher in equilibrium (27).

We use (18) to write the relative price of the routine input as a function of parameters and of the equilibrium factor price ratio:

$$\left(\frac{P_i^m}{w_i}\right)^* = \left[\left(\frac{w_i}{r_i}\right)^* A_i \alpha_i^{\frac{1}{\mu_i}}\right]^{-1} \left[1 + \left[\left(\frac{w_i}{r_i}\right)^*\right]^{-\frac{\mu}{1-\mu}} \left(\frac{1-\alpha_i}{\alpha_i}\right)^{\frac{1}{1-\mu}}\right]^{\frac{\mu_i - 1}{\mu_i}} \tag{33}$$

Next, we write the ratio of the relative price in the two countries as a function $G(\nu)$:

$$\frac{(P_1^m/w_1)^*}{(P_2^m/w_2)^*} = G(\nu) = \frac{\left[A_1 \alpha_1^{\frac{1}{\mu_1}} (\omega_2^*/\nu)\right]^{-1} \left[1 + (\omega_2^*/\nu)^{-\frac{\mu_1}{1-\mu_1}} \left(\frac{1-\alpha_1}{\alpha_1}\right)^{\frac{1}{1-\mu_1}}\right]^{\frac{\mu_1-1}{\mu_1}}}{\left[A_2 \alpha_2^{\frac{1}{\mu_2}} (\omega_2^*)\right]^{-1} \left[1 + (\omega_2^*)^{-\frac{\mu_2}{1-\mu_2}} \left(\frac{1-\alpha_2}{\alpha_2}\right)^{\frac{1}{1-\mu_2}}\right]^{\frac{\mu_2-1}{\mu_2}}} \tag{34}$$

We denote $C = \left[A_2 \alpha_2^{\frac{1}{\mu_2}} / A_1 \alpha_1^{\frac{1}{\mu_1}}\right] \left[1 + (\omega_2^*)^{-\frac{\mu_2}{1-\mu_2}} \left(\frac{1-\alpha_2}{\alpha_2}\right)^{\frac{1}{1-\mu_2}}\right]^{\frac{1-\mu_2}{\mu_2}}$ and take the derivative of the function with respect to ν to get:

$$\frac{dG(\nu)}{d\nu} = C\left[1 + (\omega_2^*/\nu)^{-\frac{\mu_1}{1-\mu_1}} \left(\frac{1-\alpha_1}{\alpha_1}\right)^{\frac{1}{1-\mu_1}}\right]^{\frac{\mu_1-1}{\mu_1}} \left\{1 - \frac{(\omega_2^*/\nu)^{-\frac{\mu_1}{1-\mu_1}} \left(\frac{1-\alpha_1}{\alpha_1}\right)^{\frac{1}{1-\mu_1}}}{1 + (\omega_2^*/\nu)^{-\frac{\mu_1}{1-\mu_1}} \left(\frac{1-\alpha_1}{\alpha_1}\right)^{\frac{1}{1-\mu_1}}}\right\}$$

The expression in curly brackets captures the two effects that ν has on the relative price of the routine input in the two countries. The positive effect works through the wage: when ν increases, the wage in the high-μ country decreases relatively to the wage in the low-μ country whereby the routine input becomes relatively more expensive in the high-μ country. The negative effect works through the price index: the price of the routine input is reduced when labor becomes cheaper. The positive effect always dominates: the derivative is always positive. Consequently, if we can characterize ν as a function of endowments and parameters and identify the value of ν at which the ratio $(P_1^m/w_1)/(P_2^m/w_2) = 1$, we can establish the range of endowments for which the high-μ country is non routine labor abundant.

We can learn more about the magnitude of ν by computing the partial derivative of the equilibrium factor price ratio with respect to μ. We apply the implicit function theorem to $F_i(\cdot)$ in (30) whereby:

$$\frac{\partial (w_i/r_i)^*}{\partial \mu} = -\frac{\partial F_i(\cdot)/\partial \mu}{\partial F_i(\cdot)/\partial (w_i/r_i)^*} \tag{35}$$

The partial derivative of $F_i(\cdot)$ with respect to the factor price ratio is negative:

$$\frac{\partial F_i(\cdot)}{\partial(w_i/r_i)^*} = -\left[\left(\frac{w_i}{r_i}\right)^*\right]^{-2}\left[c+\frac{1+c}{1-\mu}\left[\left(\frac{w_i}{r_i}\right)^*\right]^{-\frac{\mu}{1-\mu}}\left(\frac{1-\alpha_i}{\alpha_i}\right)^{\frac{1}{1-\mu}}\right]<0 \qquad (36)$$

It follows that the sign of $\partial(w_i/r_i)^*/\partial\mu$ is determined by the sign of $\partial F_i(\cdot)/\partial\mu$. Recall that the effective cost of labor is $\varpi_i = [w_i/(1-\alpha_i)]\,/\,[r_i/\alpha_i]$. We get:

$$\frac{\partial F_i(\cdot)}{\partial\mu} = -\frac{(1+c)}{(1-\mu)^2}\varpi_i^{-\frac{1}{1-\mu}}\ln\varpi_i \qquad (37)$$

We learn that labor is relatively cheap in the high-μ country when the effective cost of labor is high ($\varpi_i > 1$). Further, labor is relatively expensive in the high-μ country when the effective cost of labor is relatively low ($\varpi_i < 1$):

$$\begin{cases} \frac{\partial(w_i/r_i)^*}{\partial\mu} < 0, & \varpi_i > 1 \Leftrightarrow \frac{d\nu}{d\mu} > 0 \\ \frac{\partial(w_i/r_i)^*}{\partial\mu} = 0, & \varpi_i = 1 \Leftrightarrow \frac{d\nu}{d\mu} = 0 \\ \frac{\partial(w_i/r_i)^*}{\partial\mu} > 0, & \varpi_i < 1 \Leftrightarrow \frac{d\nu}{d\mu} < 0 \end{cases}$$

Further, we compute the derivative of the relative price with respect to the relative wage:

$$\frac{d(P_i^m/w_i)^*}{d(w_i/r_i)^*} = -\left\{\alpha_i\left[1+\left[\left(\frac{w_i}{r_i}\right)^*\right]^{-\frac{\mu}{1-\mu}}\left(\frac{1-\alpha_i}{\alpha_i}\right)^{\frac{1}{1-\mu}}\right]\right\}^{-\frac{1}{\mu}} A_i^{-1}\left[\frac{w_i}{r_i}\right]^{-2} < 0 \qquad (38)$$

Combining this derivative with our previous result on the effect of μ on the equilibrium relative wage delivers the result that the relative price of the routine input is increasing in μ whenever labor is relatively expensive.

$$\begin{cases} \frac{d(P_i^m/w_i)^*}{d(w_i/r_i)^*}\frac{\partial(w_i/r_i)^*}{\partial\mu} < 0 & \varpi_i < 1 \\ \frac{d(P_i^m/w_i)^*}{d(w_i/r_i)^*}\frac{\partial(w_i/r_i)^*}{\partial\mu} = 0 & \varpi_i = 1 \\ \frac{d(P_i^m/w_i)^*}{d(w_i/r_i)^*}\frac{\partial(w_i/r_i)^*}{\partial\mu} > 0 & \varpi_i > 1 \end{cases}$$

We learn that the factor cost channel pushes the high-μ country to specialize in non-routine production whenever labor is expensive and to specialize in routine production whenever labor is cheap. When labor is expensive, the routine input is relatively expensive in the high-μ country because labor in this country is relatively cheap, and the direct effect of the wage on the relative price of the routine input exceeds the indirect effect through which lower labor cost reduces the price of the routine input. It remains to be shown that $(P_1^m/w_1)^* = (P_2^m/w_2)^*$ when $\varpi_i = 1$ to prove that the low-μ country is non-routine labor abundant while $\varpi_i < 1$ and that the high-μ country becomes non-routine labor abundant when $\varpi_i > 1$.

3.1.5 Normalization of the CES function

We pin down the relationships between ϖ_i, ν, and factor abundance in the two countries by following Klump and De La Grandville (2000) and normalizing the CES production function. The normalization allows focusing on the structural effect of higher substitutability, e.g. the reduced incidence of decreasing marginal factor products.[6] The normalization point is defined by the level of routine production $\tilde{M}$, the capital-routine labor ratio $\tilde{\kappa} = \tilde{K}/\tilde{L}^m$ and the marginal rate of substitution $\tilde{\omega} = \tilde{w}_i/\tilde{r}_i = [(1-\alpha_i)/\alpha_i]\,\tilde{\kappa}^{1-\mu_i}$ such that at this point the capital and labor allocation to routine input production is independent of the substitutability parameter μ.

The normalized coefficient on capital α_i is:

$$\alpha_i(\mu) = \frac{\tilde{\kappa}^{1-\mu}}{\tilde{\kappa}^{1-\mu} + \tilde{\omega}} \tag{39}$$

Routine input production at the point of normalization is used to define the normalized productivity term A_i:

$$\tilde{M} = A_i(\mu)\left\{\alpha_i(\mu)\left(\tilde{K}\right)^{\mu} + [1-\alpha_i(\mu)]\left(\tilde{L}^m\right)^{\mu}\right\}^{1/\mu} \Leftrightarrow$$

$$A_i(\mu) = \frac{\tilde{M}}{\tilde{L}^m}\left[\frac{\tilde{\kappa}^{1-\mu} + \tilde{\omega}}{\tilde{\kappa} + \tilde{\omega}}\right]^{1/\mu} \tag{40}$$

We now reformulate key relationships in terms of deviation from the point of normalization. Denoting optimal factor allocation in routine input production by $\kappa_i^* = \bar{K}/L_i^{m*}$, (15) becomes:

$$\frac{\kappa_i^*}{\tilde{\kappa}} = \left[\frac{\omega_i^*}{\tilde{\omega}}\right]^{\frac{1}{1-\mu_i}} \tag{41}$$

Similarly, the implicit solution of the factor price ratio (29) becomes:

$$\omega_i^* = c\left[\frac{\bar{L}}{\bar{K}} - \frac{1+c}{\tilde{k}}\left(\frac{\omega_i^*}{\tilde{\omega}}\right)^{\frac{-1}{1-\mu_i}}\right]^{-1} \tag{42}$$

Further, the function $F(\cdot)$ in (30) becomes:

$$F_i\left(\omega_i^*;\mu_i,\frac{\bar{L}}{\bar{K}},c,\tilde{\kappa}\right) = \left(\omega_i^*\right)^{-1}c + \frac{1+c}{\tilde{\kappa}}\left[\frac{\omega_i^*}{\tilde{\omega}}\right]^{-\frac{1}{1-\mu_i}} - \frac{\bar{L}}{\bar{K}} = 0 \tag{43}$$

It is immediate from (42) that the equilibrium factor price ratio is independent of μ iff $\tilde{\omega} = \omega_i^*$. From (41) we have that $\omega_i = \tilde{\omega}$ implies $L_i^{m*}/\bar{K} = \tilde{L}_i^m/\tilde{K}$ whereby $\nu = 1$ whenever optimal factor allocation to routine input production mimicks the allocation at the point of normalization.

[6] σ is decreasing in the cross-partial derivative of production with respect to capital and labor.

3.1.6 A particular normalization: $\tilde{\kappa} = 1$

There exists one particular normalization of the CES production function for which $\varpi_i = \nu = 1$ at the point of normalization. From (39), the effective factor price ratio is $\varpi_i(\mu) = \tilde{\kappa}^{1-\mu}\left[\omega_i^*/\tilde{\omega}\right]$. Choosing $\tilde{\kappa} = 1$ at the point of normalization entails $\alpha_i = \alpha = (1+\tilde{\omega})^{-1}$, $A_i = A$, and $\varpi_i = 1$ whenever $\nu = 1$.[7] We plug these values into (42) to pin down the set of choices for initial endowments that are consistent with this normalization: $\tilde{L}/\tilde{K} > (1+c)$. We obtain the wage that for a given choice of endowments at the point of normalization equalizes the relative cost of labor in the two countries: $\tilde{\omega}_i(\tilde{L},\tilde{K};c) = c\left[(\tilde{L}/\tilde{K}) - (1+c)\right]^{-1} = \tilde{\omega}$. It is immediate that the relative price of the routine input is equalized in the two countries at the point of normalization (plug the values of α and A together with $\omega_i^* = \tilde{\omega}$ into (34)).

We now investigate how the relative wage changes when factor endowments deviate from the point of normalization. It is immediate from (42) that a shock to endowments that leaves relative endowments unchanged ($\bar{K}/\bar{L} = \tilde{K}/\tilde{L}$) leaves the relative wage unchanged and independent of μ. Thus, a proportional shock to factor endowments situates optimal routine input production on the ray from the origin to the point of normalization in the K-L_i^m plane, with factor allocation and factor prices independent of μ.

Consequently, we focus on endowment shocks that modify the capital-labor ratio in the economy relatively to the point of normalization. Without loss of generality, we fix the labor endowment $\bar{L} = \tilde{L}$ and consider shocks to the stock of capital: $\bar{K} \neq \tilde{K}$. As previously, we apply the implicit function theorem to $F(\cdot)$ in (43) to get:

$$\frac{\partial \omega_i^*}{\partial K} = -\frac{\partial F_i(\cdot)/\partial K}{\partial F_i(\cdot)/\partial \omega_i^*} > 0 \tag{44}$$

An increase (decrease) in the capital stock unambiguously increases (decreases) the relative wage ω_i^*. Consequently, the relative wage exceeds the relative wage at the point of normalization whenever the stock of capital exceeds the stock of capital at the point of normalization:

$$\begin{cases} \frac{\omega_i^*}{\tilde{\omega}} > 1, & \bar{K} > \tilde{K} \\ \frac{\omega_i^*}{\tilde{\omega}} = 1, & \bar{K} = \tilde{K} \\ \frac{\omega_i^*}{\tilde{\omega}} < 1, & \bar{K} < \tilde{K} \end{cases}$$

We have previously established that the relative wage is decreasing in μ whenever the effective cost of labor ϖ_i^* exceeds 1. With this normalization, $\varpi_i = \omega_i^*/\tilde{\omega}$. Hence, we reformulate

[7] For simplicity, we can always normalize $A = 1$ by defining $\tilde{M} = \tilde{L}^m = \tilde{K}$.

our results in terms of shocks to endowments relatively to the point of normalization:

$$\begin{cases} \frac{\partial \omega_i^*}{\partial \mu} < 0, & \bar{K} > \tilde{K} \;\Leftrightarrow\; \frac{d\nu}{d\mu} > 0 \\ \frac{\partial \omega_i^*}{\partial \mu} = 0, & \bar{K} = \tilde{K} \;\Leftrightarrow\; \frac{d\nu}{d\mu} = 0 \\ \frac{\partial \omega_i^*}{\partial \mu} > 0, & \bar{K} < \tilde{K} \;\Leftrightarrow\; \frac{d\nu}{d\mu} < 0 \end{cases}$$

To sum up, a higher μ dampens the effect of any shock to factor endowments on the equilibrium relative wage.[8] Thus, if the shock to the stock of capital is positive, labor becomes more expensive than at the point of normalization in both countries, but less so in the high-μ country: $\tilde{\omega} < \omega_1^* < \omega_2^*$. If the shock to the stock of capital is negative, labor becomes less expensive than at the point of normalization in both countries, but less so in the high-μ country: $\tilde{\omega} > \omega_1^* > \omega_2^*$. This dampening effect leads the high-μ country to specialize in non-routine production when labor becomes relatively scarce (through capital deepening) and to specialize in routine production when labor becomes relatively abundant. Indeed, the relative price of the routine input is increasing (decreasing) in μ whenever the stock of capital increases (decreases) relatively to the point of normalization:

$$\begin{cases} \frac{d(P_i^m/w_i)^*}{d\omega_i^*}\frac{\partial \omega_i^*}{\partial \mu} < 0 & \bar{K} < \tilde{K} \\ \frac{d(P_i^m/w_i)^*}{d\omega_i^*}\frac{\partial \omega_i^*}{\partial \mu} = 0 & \bar{K} = \tilde{K} \\ \frac{d(P_i^m/w_i)^*}{d\omega_i^*}\frac{\partial \omega_i^*}{\partial \mu} > 0 & \bar{K} > \tilde{K} \end{cases}$$

This result suffices to establish that the high-μ country is relatively non-routine labor abundant (32) under capital deepening: $(L_{a1}/S_1)^* > (L_{a2}/S_2)^*$.[9] The intuition behind this result is the following. Consider a shock to technology in the low-μ country such that μ increases. When μ goes up, the same quantity of inputs delivers more output in the routine sector $(M_1(w_2/r_2) > M_2(w_2/r_2)$. But labor is expensive relatively to the cost-minimizing factor combination in routine production because of the increase in μ. Consequently, labor is released from routine tasks, and this labor can only be absorbed in non-routine tasks whereby $M_1(\omega_2^*) \gg M_1(\omega_1^*)$. The price of labor goes down up to the point where extra labor absorbed in production of final goods is just enough to absorb excess labor released from routine input production. It follows that $L_1^{a*} \gg L_2^{a*}$. The ambiguity comes from the fact that the release of labor from routine input production does not suffice to prove that $M_1^* \leq M_2^*$. The high-μ country becomes non-routine abundant because the direct effect on labor allocation outweighs the indirect effect on routine input production: $L_1^{a*}/M_1^* \gg L_2^{a*}/M_2^*$.

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8 Any given change in capital intensity leads to a smaller change in the marginal product of labor in the high-μ country because μ is inversely related to the cross-partial derivative of output with respect to K and L.

9 This statement is equivalent to saying that the non-routine intensive good is relatively cheap in the high-μ country: $P_{11}/P_{12} < P_{21}/P_{22}$.

3.1.7 Generalization: $\kappa \neq 1$

More generally, we can choose any $\kappa \neq 1$ at the point of normalization whereby $\varpi_i = 1$ when $\omega_i^* = \tilde{\omega}\tilde{\kappa}^{\mu_i - 1}$ and $\varpi_i \neq 1$ at the point of normalization defined by $\nu = 1 \Leftrightarrow \omega_i^* = \tilde{\omega}$. The distribution and productivity terms are now country-specific. We plug these values into (42) to pin down the set of feasible choices for initial endowments: $\tilde{L}/\tilde{K} > (1+c)/\tilde{\kappa}$. We obtain the wage that equalizes the relative wage in the two countries for a given choice of endowments at the point of normalization: $\tilde{\omega}(\tilde{L}, \tilde{K}; c) = c\left[(\tilde{L}/\tilde{K}) - (1+c)/\tilde{\kappa}\right]^{-1}$.[10] As previously, the relative price of the routine input is equalized in the two countries at the point of normalization (plug the values of α_i and A_i together with $\omega_i^* = \tilde{\omega}$ into (34)).

Again, we investigate how the relative wage changes when we deviate from the point of normalization. The derivative of the equilibrium wage with respect to the capital stock is positive whereby the following relationships continue to hold:

$$\begin{cases} \frac{\omega_i^*}{\tilde{\omega}} > 1, & \bar{K} > \tilde{K} \\ \frac{\omega_i^*}{\tilde{\omega}} = 1, & \bar{K} = \tilde{K} \\ \frac{\omega_i^*}{\tilde{\omega}} < 1, & \bar{K} < \tilde{K} \end{cases}$$

Further, the sign of $\partial(w_i/r_i)^*/\partial\mu$ is still determined by the sign of $\partial F_i(\cdot)/\partial\mu$ because $\partial F(\cdot)/\partial\omega_i^* < 0$. The latter is now directly determined by the wage relatively to the wage at the point of normalization:

$$\frac{\partial F_i(\cdot)}{\partial\mu} = -\ln\left(\frac{\omega_i^*}{\tilde{\omega}}\right)\frac{(1+c)}{\tilde{\kappa}(1-\mu)^2}\left[\frac{\omega_i^*}{\tilde{\omega}}\right]^{-\frac{1}{1-\mu}} \tag{45}$$

Thus, labor is relatively cheap in the high-μ country when the cost of labor increases relatively to the point of normalization. Further, labor is relatively expensive in the high-μ country when the cost of labor decreases relatively to the point of normalization:

$$\begin{cases} \frac{\partial\omega_i^*}{\partial\mu} < 0, & \left(\frac{\omega_i^*}{\tilde{\omega}}\right) > 1 \Leftrightarrow \bar{K} > \tilde{K} \Leftrightarrow \frac{d\nu}{d\mu} > 0 \\ \frac{\partial\omega_i^*}{\partial\mu} = 0, & \left(\frac{\omega_i^*}{\tilde{\omega}}\right) = 1 \Leftrightarrow \bar{K} = \tilde{K} \Leftrightarrow \frac{d\nu}{d\mu} = 0 \\ \frac{\partial\omega_i^*}{\partial\mu} > 0, & \left(\frac{\omega_i^*}{\tilde{\omega}}\right) < 1 \Leftrightarrow \bar{K} < \tilde{K} \Leftrightarrow \frac{d\nu}{d\mu} < 0 \end{cases}$$

A higher μ dampens the effect of any shock to factor endowments on the equilibrium relative wage.[11] Thus, if the shock to the stock of capital is positive, labor becomes more expensive than

[10] Equivalently, $\tilde{\omega} = c\tilde{K}\left[\tilde{L} - (1+c)\tilde{L}^m\right]^{-1}$.

[11] Any given change in capital intensity leads to a smaller change in the marginal product of labor in the high-μ country because μ is inversely related to the cross-partial derivative of output with respect to K and L.

at the point of normalization in both countries, but less so in the high-μ country: $\tilde{\omega} < \omega_1^* < \omega_2^*$. If the shock to the stock of capital is negative, labor becomes less expensive than at the point of normalization in both countries, but less so in the high-μ country: $\tilde{\omega} > \omega_1^* > \omega_2^*$.

This dampening effect leads the high-μ country to specialize in non-routine production when labor becomes relatively scarce (through capital deepening) and to specialize in routine production when labor becomes relatively abundant. Indeed, the relative price of the routine input is increasing (decreasing) in μ whenever the stock of capital increases (decreases) relatively to the point of normalization:

$$\begin{cases} \frac{d(P_i^m/w_i)^*}{d\omega_i^*}\frac{\partial \omega_i^*}{\partial \mu} < 0 & \bar{K} < \tilde{K} \\ \frac{d(P_i^m/w_i)^*}{d\omega_i^*}\frac{\partial \omega_i^*}{\partial \mu} = 0 & \bar{K} = \tilde{K} \\ \frac{d(P_i^m/w_i)^*}{d\omega_i^*}\frac{\partial \omega_i^*}{\partial \mu} > 0 & \bar{K} > \tilde{K} \end{cases}$$

To sum up, the choice of $\tilde{\kappa}$ has no incidence on the mechanism at work. To simplify notation, we will henceforth work with the specific case of $\tilde{\kappa} = 1$.

3.2 The PPF

Final good production is:

$$Q_1 = [L_1 - L_1^m]^{1-\beta_1} [\alpha K_1{}^{\mu} + (1-\alpha)(L_1^m)^{\mu}]^{\beta_1/\mu}$$
$$Q_2 = [L_2 - L_2^m]^{1-\beta_2} [\alpha K_2{}^{\mu} + (1-\alpha)(L_2^m)^{\mu}]^{\beta_2/\mu}$$

Resource constraints are:

$$L_1 + L_2 = \bar{L}$$
$$K_1 + K_2 = \bar{K}$$

We can simplify the expressions by incorporating the fact that the factor use ratio of capital and routine labor must be equalized in routine production: $MRTS_{M_1} = MRTS_{M_2}$ delivers $L_1^m/K_1 = L_2^m/K_2$. Using $K_1 + K_2 = \bar{K}$ and $L_1^m + L_2^m = L^m$, we get $L_1^m/K_1 = L_2^m/K_2 = L^m/\bar{K}$. We can rewrite routine production in each sector as:

$$M_1 = K_1 [\alpha + (1-\alpha)(L^m/\bar{K})^{\mu}]^{1/\mu}$$
$$M_2 = K_2 [\alpha + (1-\alpha)(L^m/\bar{K})^{\mu}]^{1/\mu}$$
$$M = M_1 + M_2 = [\alpha \bar{K}^{\mu} + (1-\alpha)(L^m)^{\mu}]^{1/\mu}$$

Final good production becomes:

$$Q_1 = [L_1^a]^{1-\beta_1} K_1^{\beta_1} [\alpha + (1-\alpha)(L^m/\bar{K})^{\mu}]^{\beta_1/\mu}$$
$$Q_2 = [L_2^a]^{1-\beta_2} K_2^{\beta_2} [\alpha + (1-\alpha)(L^m/\bar{K})^{\mu}]^{\beta_2/\mu}$$

Resource constraints become:

$$M = [\alpha \bar{K}^{\mu} + (1-\alpha)(L^m)^{\mu}]^{1/\mu}$$
$$L^m + L_1^a + L_2^a = \bar{L}$$
$$K_1 + K_2 = \bar{K}$$

Incorporating resource constraints in production, we get:

$$Q_1 = [\bar{L} - L_m - L_2^a]^{1-\beta_1} [\bar{K} - K_2]^{\beta_1} [\alpha + (1-\alpha)(L^m/\bar{K})^{\mu}]^{\beta_1/\mu}$$
$$Q_2 = [L_2^a]^{1-\beta_2} K_2^{\beta_2} [\alpha + (1-\alpha)(L^m/\bar{K})^{\mu}]^{\beta_2/\mu}$$

We maximize Q_1 by choosing $\{L_m, L_2^a, K_2\}$ subject to $Q_2 \leq L_2^a \left[K_2/L_2^a\right]^{\beta_2} [\alpha + (1-\alpha)(L^m/\bar{K})^{\mu}]^{\beta_2/\mu}$. The Lagrangian for the constrained maximization problem is:

$$\mathscr{L} = [\bar{L} - L_m - L_2^a]^{1-\beta_1} [\bar{K} - K_2]^{\beta_1} [\alpha + (1-\alpha)(L^m/\bar{K})^\mu]^{\beta_1/\mu} -$$
$$\lambda \left\{ (L_2^a)^{1-\beta_2} K_2^{\beta_2} [\alpha + (1-\alpha)(L^m/\bar{K})^\mu]^{\beta_2/\mu} - \bar{Q}_2 \right\}$$

Three first order conditions mimick the standard HO model, with an additional condition that pins down the choice of labor allocation to routine and non-routine tasks.

The first FOC pins down the impact of reallocating abstract labor from good 1 to good 2 without changing labor allocation between routine and non-routine tasks.

$$\frac{\partial \mathscr{L}}{\partial L_2^a} = -(1-\beta_1) \left[\frac{\bar{K} - K_2}{\bar{L} - L_m - L_2^a} \right]^{\beta_1} [\alpha + (1-\alpha)(L^m/\bar{K})^\mu]^{\beta_1/\mu} -$$
$$\lambda (1-\beta_2) \left[\frac{K_2}{L_2^a} \right]^{\beta_2} [\alpha + (1-\alpha)(L^m/\bar{K})^\mu]^{\beta_2/\mu} = 0$$

The second FOC pins down the impact of reallocating capital from good 1 to good 2 without changing the allocation of labor to routine and non-routine tasks and without changing the allocation of abstract labor to goods.[12]

$$\frac{\partial \mathscr{L}}{\partial K_2} = -\beta_1 \left[\frac{\bar{K} - K_2}{\bar{L} - L_m - L_2^a} \right]^{\beta_1 - 1} [\alpha + (1-\alpha)(L^m/\bar{K})^\mu]^{\beta_1/\mu} -$$
$$\lambda \beta_2 \left[\frac{K_2}{L_2^a} \right]^{\beta_2 - 1} [\alpha + (1-\alpha)(L^m/\bar{K})^\mu]^{\beta_2/\mu} = 0$$

The third FOC pins down the impact of reallocating labor between routine and non-routine tasks. It thus adds the efficiency requirement specific to our model that the marginal productivity of labor in routine and non-routine tasks must be equalized in the production of both goods.

$$\frac{\partial \mathscr{L}}{\partial L^m} = \left[\frac{\bar{K} - K_2}{\bar{L} - L_m - L_2^a} \right]^{\beta_1} \left[\alpha + (1-\alpha) \left(\frac{L^m}{\bar{K}} \right)^\mu \right]^{\frac{\beta_1}{\mu}} \left\{ (\beta_1 - 1) + \beta_1 (1-\alpha) \frac{(L^m/\bar{K})^\mu}{L^m} \frac{\bar{L} - L_m - L_2^a}{\alpha + (1-\alpha)(L^m/\bar{K})^\mu} \right\}$$
$$-\lambda \beta_2 (1-\alpha) \frac{(L^m/\bar{K})^\mu}{L^m} \left[\frac{K_2}{L_2^a} \right]^{\beta_2} \left[\alpha + (1-\alpha) \left(\frac{L^m}{\bar{K}} \right)^\mu \right]^{\frac{\beta_2}{\mu}} \frac{L_2^a}{\alpha + (1-\alpha)(L^m/\bar{K})^\mu} = 0$$

The last FOC states that Q_1 is maximized for given Q_2: $\bar{Q}_2 = \left[L_2^a \right]^{1-\beta_2} K_2^{\beta_2} [\alpha + (1-\alpha)(L^m/\bar{K})^\mu]^{\beta_2/\mu}$.

We rearrange the latter condition to solve for the share of capital allocated to sector 2:

$$[1] \qquad \frac{K_2}{\bar{K}} = \left[\bar{Q}_2 \right]^{\frac{1}{\beta_2}} [L_2^a]^{\frac{\beta_2 - 1}{\beta_2}} [\alpha \bar{K}^\mu + (1-\alpha)(L^m)^\mu]^{\frac{-1}{\mu}}$$

[12] Reallocating capital from good 1 to good 2 means that routine labor is reallocated to good 2 in the same proportion (so that $MRTS_{M_1} = MRTS_{M_2}$ continues to hold.

Combining FOC-1 and FOC-2, we get the optimality condition $MRTS_{Q1} = MRTS_{Q2}$:

$$\frac{(1-\beta_1)}{\beta_1}\frac{(\bar{K}-K_2)}{(\bar{L}-L_m-L_2^a)} = \frac{(1-\beta_2)}{\beta_2}\frac{K_2}{L_2^a}$$

This condition can be rearranged to solve for the share of capital allocated to sector 2 as a function of the share of abstract labor allocated to sector 2:

$$[2] \qquad \frac{K_2}{\bar{K}} = \frac{(1-\beta_1)\beta_2}{(\beta_2-\beta_1)+\beta_1(1-\beta_2)\frac{\bar{L}-L^m}{L_2^a}} = \frac{(1-\beta_1)\beta_2 L_2^a}{(\beta_2-\beta_1)L_2^a+\beta_1(1-\beta_2)(\bar{L}-L^m)}$$

$$\frac{K_2}{\bar{K}} = \frac{(1-\beta_1)\beta_2 L_2^a}{\beta_1\left[\bar{L}-L^m-L_2^a\right]+\beta_2 L_2^a-\beta_1\beta_2\left[\bar{L}-L^m\right]}$$

Combining [1] and [2], the first expression for L_2^a and L^m as a function of $\bar{Q}_2$ is:

$$\left[\bar{Q}_2\right]^{\frac{1}{\beta_2}}\left[L_2^a\right]^{\frac{\beta_2-1}{\beta_2}}\left[\alpha\bar{K}^\mu+(1-\alpha)(L^m)^\mu\right]^{\frac{-1}{\mu}} = \frac{(1-\beta_1)\beta_2 L_2^a}{\beta_1\left[\bar{L}-L^m-L_2^a\right]+\beta_2 L_2^a-\beta_1\beta_2\left[\bar{L}-L^m\right]}$$

Rearranging gives the first equation of the system we need to solve for $\left\{L^m, L_2^a\right\}$:

$$\{1\} \quad \left[\bar{Q}_2\right]^{\frac{1}{\beta_2}}\left[\beta_1\left(\bar{L}-L^m-L_2^a\right)+\beta_2 L_2^a-\beta_1\beta_2\left(\bar{L}-L^m\right)\right] = (1-\beta_1)\beta_2\left[L_2^a\right]^{\frac{1}{\beta_2}}\left[\alpha\bar{K}^\mu+(1-\alpha)(L^m)^\mu\right]^{\frac{1}{\mu}}$$

Further, combine FOC-1 with FOC-3 to get:

$$\frac{\beta_1-1}{(\beta_1-1)+\beta_1(1-\alpha)\frac{(L^m/\bar{K})^\mu}{L^m}\frac{\bar{L}-L_m-L_2^a}{\alpha+(1-\alpha)(L^m/\bar{K})^\mu}} = \frac{1-\beta_2}{\beta_2(1-\alpha)\frac{(L^m/\bar{K})^\mu}{L^m}\frac{L_2^a}{\alpha+(1-\alpha)(L^m/\bar{K})^\mu}}$$

Rearranging gives the second equation of the system we need to solve for $\left\{L^m, L_2^a\right\}$:

$$\begin{aligned}\{2\} \qquad (1-\beta_1)\left\{(1-\beta_2)\alpha L^m+(1-\alpha)\left(L^m/\bar{K}\right)^\mu\left[(1-\beta_2)L^m-\beta_2 L_2^a\right]\right\} &= \\ (1-\beta_2)\beta_1(1-\alpha)\left(L^m/\bar{K}\right)^\mu\left[\bar{L}-L_m-L_2^a\right]&\end{aligned}$$

Solution mechanism: pick endowments and parameters. Solve for $\max(Q_2)$ when $Q_1 = 0$ and solve for $\max(Q_1)$ when $Q_2 = 0$ (in these cases, problem is to optimally allocate L between L^m and L^a in good 1 or good 2). Next, consider a grid of $\bar{Q}_2$ values from $Q_2 = 0$ to $\max(Q_2)$. Given choice of $\bar{Q}_2$, solve system $\{1\},\{2\}$ for L^m and L_2^a, deduce the other variables to solve for Q_1.

3.3 Opening up to trade

3.3.1 The intuition

Opening up to trade amplifies differences in labor allocation to routine and non-routine tasks that were observed in autarky. The intuition is the following. Differences in capital-labor substitutability in the two countries lead to a wedge in the MPL/MPK ratio in the autarky equilibrium which leads to a wedge in the relative autarky price of the two final goods. When the relative wage increases, the cost of labor allocated to non-routine tasks increases by more than the cost of the routine input because the latter uses both capital and labor. Consequently, the final good that requires more labor in non-routine tasks is relatively cheap in the country with the relatively low MPL/MPK ratio.

Trade equalizes the relative price of the two final goods by increasing the relative price of the good that was relatively cheap in autarky. The MPL/MPK ratio decreases in the country where it was relatively high, and it increases in the country where it was relatively low. The capital endowment is fixed by assumption. It follows that the only way to reduce (increase) the MPL/MPK ratio is to move labor into (out of) routine input production. Thus, the country that had a relatively high MPL/MPK ratio and, consequently, a relatively low price of the routine-intensive good, allocates more labor to routine input production. At the same time, the country that had a relatively low MPL/MPK ratio allocates more labor to non-routine tasks.

3.3.2 The illustration

The price of the final good is:

$$P_{ig} = \frac{w_i^{1-\beta_g} P_i^{m\beta_g}}{z_g \beta_g^{\beta_g}(1-\beta_g)^{1-\beta_g}}$$

We replace P_i^m by its value and rearrange the expression to get:

$$P_{ig} = \frac{w_i \left(\frac{w_i}{r_i}\right)^{-\beta_g} \alpha_i^{-\frac{\beta_g}{\mu_i}} \left[1+\left(\frac{w_i}{r_i}\right)^{-\frac{\mu_i}{1-\mu_i}} \left(\frac{1-\alpha_i}{\alpha_i}\right)^{\frac{1}{1-\mu_i}}\right]^{-\frac{\beta_g(1-\mu_i)}{\mu_i}}}{A_i^{\beta_g} z_g \beta_g^{\beta_g}(1-\beta_g)^{1-\beta_g}}$$

The relative price of the two final goods is:

$$\frac{P_{i1}}{P_{i2}} = \frac{z_2 \beta_2^{\beta_2}(1-\beta_2)^{1-\beta_2}}{z_1 \beta_1^{\beta_1}(1-\beta_1)^{1-\beta_1} \alpha_i^{\frac{\beta_1-\beta_2}{\mu_i}} \left(\frac{w_i}{r_i}\right)^{\beta_1-\beta_2} A_i^{\beta_1-\beta_2} \left[1+\left(\frac{w_i}{r_i}\right)^{-\frac{\mu_i}{1-\mu_i}} \left(\frac{1-\alpha_i}{\alpha_i}\right)^{\frac{1}{1-\mu_i}}\right]^{\frac{(\beta_1-\beta_2)(1-\mu_i)}{\mu_i}}}$$

To simplify the expression, we use the normalization $\tilde{\kappa} = 1$ whereby $A_i = 1$ and $\alpha_i = (1+\tilde{\omega})^{-1}$ and further group all the country-invariant terms under the constant B. We have:

$$\frac{P_{i1}}{P_{i2}} = B(1+\tilde{\omega})^{\frac{\beta_1-\beta_2}{\mu_i}} \left\{ \omega_i \left[1+\omega_i \left(\frac{\omega_i}{\tilde{\omega}} \right)^{-\frac{1}{1-\mu_i}} \right]^{\frac{(1-\mu_i)}{\mu_i}} \right\}^{\beta_2-\beta_1}$$

Introducing ω_i into square brackets we get:

$$\frac{P_{i1}}{P_{i2}} = B(1+\tilde{\omega})^{\frac{\beta_1-\beta_2}{\mu_i}} \left[\omega_i^{\frac{\mu_i}{1-\mu_i}} + \tilde{\omega}^{\frac{1}{1-\mu_i}} \right]^{\frac{(\beta_2-\beta_1)(1-\mu_i)}{\mu_i}}$$

The derivative of the relative price wrt the relative wage ω_i is positive if good 1 is non-routine abundant ($\beta_1 < \beta_2$). Next, consider the relative price of the two final goods for the two countries:

$$\frac{P_{11}/P_{12}}{P_{21}/P_{22}} = (1+\tilde{\omega})^{\frac{(\mu_1-\mu_2)(\beta_2-\beta_1)}{\mu_1\mu_2}} \left[\omega_1^{\frac{\mu_1}{1-\mu_1}} + \tilde{\omega}^{\frac{1}{1-\mu_1}} \right]^{\frac{(\beta_2-\beta_1)(1-\mu_1)}{\mu_1}} \left[\omega_2^{\frac{\mu_2}{1-\mu_2}} + \tilde{\omega}^{\frac{1}{1-\mu_2}} \right]^{\frac{(\beta_1-\beta_2)(1-\mu_2)}{\mu_2}}$$

We use $\omega_2/\omega_1 = \nu$ to write:

$$\frac{P_{11}/P_{12}}{P_{21}/P_{22}} = (1+\tilde{\omega})^{\frac{(\mu_1-\mu_2)(\beta_2-\beta_1)}{\mu_1\mu_2}} \left[(\omega_2/\nu)^{\frac{\mu_1}{1-\mu_1}} + \tilde{\omega}^{\frac{1}{1-\mu_1}} \right]^{\frac{(\beta_2-\beta_1)(1-\mu_1)}{\mu_1}} \left[\omega_2^{\frac{\mu_2}{1-\mu_2}} + \tilde{\omega}^{\frac{1}{1-\mu_2}} \right]^{\frac{(\beta_1-\beta_2)(1-\mu_2)}{\mu_2}}$$

The above expression illustrates that any change in the relative price ratio can be studied as a function of the wedge in the relative wage of country 2 and country 1. It is immediate that the relative price of the non-routine intensive good is decreasing in ν.

Suppose $\nu > 1$ in autarky. To equate the relative price of the non-routine intensive good in both countries, ν must be reduced whereby ω_1 must go up. The latter can only occur if we move labor out of routine input production in country 1. Hence, country 1 specializes in the non-routine intensive good when the relative autarky price of this good is lower in country 1. Suppose $\nu < 1$ in autarky. To equate the relative price of the non-routine intensive good in both countries, ν must increase whereby ω_2 must go up. The latter can only occur if we move labor out of routine input production in country 2. Hence, country 2 specializes in the non-routine intensive good when the relative autarky price of this good is lower in country 2.

3.3.3 Free Trade Equilibrium

The free trade equilibrium is a vector of allocations for consumers ($\hat{Q}_{ig}, i, g = 1,2$), allocations for the firm ($\hat{K}_{ig}$, $\hat{L}^m_{ig}$, $\hat{L}^a_{ig}$, $\hat{M}_{ig}, i, g = 1,2$), and prices ($\hat{w}_i$, $\hat{r}_i$, $\hat{P}^m_i$, $\hat{P}_g, i, g = 1,2$) such that given prices consumer's allocation maximizes utility, and firms' allocations solve the cost minimization problem in each country, goods and factor markets clear: $\sum_i \hat{Q}_{ig} = \sum_i \hat{Y}_{ig}, g = 1,2$; $\sum_g \hat{K}_{ig} = \bar{K}, i = 1,2$; $\sum_g \hat{L}^a_{ig} + \hat{L}^m_{ig} = \bar{L}, i = 1,2$; $\sum_g \hat{M}_{ig} = \hat{M}_i, i = 1,2$.

Whenever both final goods are produced in both countries, firms' allocations satisfy: $\beta_g P_g z_g M_{ig}^{\beta_g-1} L_{ig}^{a\ 1-\beta_g} = P_i^m$ and $(1-\beta_g) P_g z_g M_{ig}^{\beta_g} L_{ig}^{a\ -\beta_g} = w_i$. Further, from the ZPC, the price of each final good in each country is $P_{ig} = P_i^{m\beta_g} w_i^{1-\beta_g}/Z$ where $Z = z_g \beta_g^{\beta_g}(1-\beta_g)^{1-\beta_g}$. Prices are equalized through trade whereby: $(P_1^m/P_2^m)^{\beta_g} = (w_2/w_1)^{1-\beta_g}$. We solve for P_1^m/P_2^m in one sector and plug the solution in the expression for the other sector to get:

$$\left(\frac{w_2}{w_1}\right)^{\frac{1-\beta_2}{\beta_2}} = \left(\frac{w_2}{w_1}\right)^{\frac{1-\beta_1}{\beta_1}}, \beta_2 \neq \beta_1 \;\Leftrightarrow\; w_2 \;=\; w_1 \tag{46}$$

As in the canonical HO model, trade leads to factor price equalization: the cost of labor and the cost of the routine input are equalized through trade. The feature specific to our model is that in general opening up to trade does not result in capital cost equalization. To see why, recall that firms' cost minimization in routine input production delivers (19). Given FPE, we have: $P^m = A_i^{-1}\left[\alpha_i^{\sigma_i} r_i^{1-\sigma_i} + (1-\alpha_i)^{\sigma_i} w^{1-\sigma_i}\right]^{\frac{1}{1-\sigma_i}}$. We use the normalization $\tilde{\kappa} = 1$ whereby $A_i = 1$ and $\alpha_i = (1+\tilde{\omega})^{-1}$ to simplify this expression and to solve for r_i in each country:

$$\begin{cases} r_1 = \left[(1+\tilde{\omega})^{\sigma_1} P^{m\,1-\sigma_1} - \tilde{\omega}^{\sigma_1} w^{1-\sigma_1}\right]^{\frac{1}{1-\sigma_1}} \\ r_2 = \left[(1+\tilde{\omega})^{\sigma_2} P^{m\,1-\sigma_2} - \tilde{\omega}^{\sigma_2} w^{1-\sigma_2}\right]^{\frac{1}{1-\sigma_2}} \end{cases}$$

The two expressions only differ by μ whereby in general $r_1 \neq r_2$.[13] Below we show that $r_1 = r_2$ iff $w/r_1 = w/r_2 = \tilde{\omega}$.

We connect the equilibrium relative price of the routine input and of labor to the allocation of resources to routine and non-routine tasks. Firm cost minimization in final goods' production delivers $\beta_g P_g z_g M_{ig}^{\beta_g}(L_{ig}^a)^{1-\beta_g} = P^m M_{ig}$ and $(1-\beta_g) P_g z_g M_{ig}^{\beta_g}(L_{ig}^a)^{1-\beta_g} = w L_{ig}^a$. Rearranging these two expressions and summing across countries delivers:

$$\begin{cases} P_g Y_{ig} = P^m M_{ig}/\beta_g \;\Leftrightarrow\; \sum_i Y_{ig} = \frac{P^m}{P_g \beta_g} \sum_i M_{ig} \\ P_g Y_{ig} = w L_{ig}^a/(1-\beta_g) \;\Leftrightarrow\; \sum_i Y_{ig} = \frac{w}{P_g(1-\beta_g)} \sum_i L_{ig}^a \end{cases}$$

First order conditions of the consumer problem in each country give:

$$\begin{cases} \theta_1 = \lambda_i P_1 Q_{i1} \\ \theta_2 = \lambda_i P_2 Q_{i2} \end{cases}$$

Summing the FOCs for each good in the two countries gives: $\theta_g/\lambda_1 + \theta_g/\lambda_2 = P_g(\sum_i Q_{ig})$. From goods' market clearing $\sum_i Q_{ig} = \sum_i Y_{ig}$. Plugging in the two expressions of $\sum_i Y_{ig}$ we get:

[13] Expressions are more cumbersome if the general normalization $\kappa \neq 1$ is used, but the conclusion is unchanged.

$$\begin{cases} \sum_i \frac{\theta_g}{\lambda_i} = \frac{P^m}{\beta_g} \sum_i M_{ig} \Leftrightarrow \sum_i \frac{1}{\lambda_i} \sum_g \beta_g \theta_g = P^m \sum_g \sum_i M_{ig} \Leftrightarrow \sum_i \frac{1}{\lambda_i} = P^m \frac{M_1^* + M_2^*}{\sum_g \beta_g \theta_g} \\ \sum_i \frac{\theta_g}{\lambda_i} = \frac{w}{(1-\beta_g)} \sum_i L_{ig}^a \Leftrightarrow \sum_i \frac{1}{\lambda_i} \sum_g (1-\beta_g)\theta_g = P^m \sum_g \sum_i L_{ig}^a \Leftrightarrow \sum_i \frac{1}{\lambda_i} = w \frac{L_1^{a*} + L_2^{a*}}{\sum_g (1-\beta_g)\theta_g} \end{cases}$$

Combining the above expressions delivers:

$$\frac{L_1^{a*} + L_2^{a*}}{M_1^* + M_2^*} = c\frac{P^m}{w} \tag{47}$$

Notice that the expression on the RHS can be written in two ways, depending on whether we use the expression of the price index in country 1 or in country 2. Replacing P^m by its value in each of the two countries gives:

$$A_1^{-1}\left[\alpha_1^{\frac{1}{1-\mu_1}}\left(\frac{w}{r_1}\right)^{\frac{\mu_1}{1-\mu_1}} + (1-\alpha_1)^{\frac{1}{1-\mu_1}}\right]^{-\frac{1-\mu_1}{\mu_1}} = A_2^{-1}\left[\alpha_2^{\frac{1}{1-\mu_2}}\left(\frac{w}{r_2}\right)^{\frac{\mu_2}{1-\mu_2}} + (1-\alpha_2)^{\frac{1}{1-\mu_2}}\right]^{-\frac{1-\mu_2}{\mu_2}}$$

We use the normalization $\tilde{\kappa} = 1$ whereby $A_i = 1$ and $\alpha_i = (1+\tilde{\omega})^{-1}$ to get:

$$(1+\tilde{\omega})^{\frac{1}{\mu_1}}\left[\left(\frac{w}{r_1}\right)^{\frac{\mu_1}{1-\mu_1}} + \tilde{\omega}^{\frac{1}{1-\mu_1}}\right]^{-\frac{1-\mu_1}{\mu_1}} = (1+\tilde{\omega})^{\frac{1}{\mu_2}}\left[\left(\frac{w}{r_2}\right)^{\frac{\mu_2}{1-\mu_2}} + \tilde{\omega}^{\frac{1}{1-\mu_2}}\right]^{-\frac{1-\mu_2}{\mu_2}} \tag{48}$$

It is easy to check that setting $w/r_1 = w/r_2 = \tilde{\omega}$ solves (48). As expected, at the point of normalization, resource allocation and equilibrium relative factor prices are the same in both countries. In all other cases we can solve for the equilibrium factor price ratio in one country as a function of the factor price ratio in the other country:

$$\frac{w}{r_1} = \left\{(1+\tilde{\omega})^{\frac{\mu_2-\mu_1}{\mu_2(1-\mu_1)}}\left[\left(\frac{w}{r_2}\right)^{\frac{\mu_2}{1-\mu_2}} + \tilde{\omega}^{\frac{1}{1-\mu_2}}\right]^{\frac{\mu_1(1-\mu_2)}{\mu_2(1-\mu_1)}} - \tilde{\omega}^{\frac{1}{1-\mu_1}}\right\}^{\frac{1-\mu_1}{\mu_1}}$$

$$\omega_1^* = \left\{(1+\tilde{\omega})^{\frac{\sigma_2-\sigma_1}{\sigma_2-1}}\left[\left(\frac{w}{r_2}\right)^{\sigma_2-1} + \tilde{\omega}^{\sigma_2}\right]^{\frac{\sigma_1-1}{\sigma_2-1}} - \tilde{\omega}^{\sigma_1}\right\}^{\frac{1}{\sigma_1-1}} \tag{49}$$

$$\frac{w}{r_2} = \left\{(1+\tilde{\omega})^{\frac{\mu_1-\mu_2}{\mu_1(1-\mu_2)}}\left[\left(\frac{w}{r_1}\right)^{\frac{\mu_1}{1-\mu_1}} + \tilde{\omega}^{\frac{1}{1-\mu_1}}\right]^{\frac{\mu_2(1-\mu_1)}{\mu_1(1-\mu_2)}} - \tilde{\omega}^{\frac{1}{1-\mu_2}}\right\}^{\frac{1-\mu_2}{\mu_2}}$$

$$\omega_2^* = \left\{(1+\tilde{\omega})^{\frac{\sigma_1-\sigma_2}{\sigma_1-1}}\left[\left(\frac{w}{r_1}\right)^{\sigma_1-1} + \tilde{\omega}^{\sigma_1}\right]^{\frac{\sigma_2-1}{\sigma_1-1}} - \tilde{\omega}^{\sigma_2}\right\}^{\frac{1}{\sigma_2-1}} \tag{50}$$

Next, we work with the LHS of (47). We use firm cost minimization in routine input production together with factor market clearing to rewrite the LHS as a function of the equilibrium factor price ratio and factor endowments. Capital market clearing (see 70) delivers:

$$M_i^* = A_i \alpha_i^{1/\mu_i} \bar{K} \left[1 + (\omega_i^*)^{-\frac{\mu_i}{1-\mu_i}} \left(\frac{1-\alpha_i}{\alpha_i} \right)^{\frac{1}{1-\mu_i}} \right]^{1/\mu_i} \tag{51}$$

Labor market clearing delivers $L_i^{a*} = \bar{L} - L_i^{m*}$ while cost minimization in routine input production and the total capital stock determine labor allocation to routine tasks:

$$L_i^{m*}(M_i^*) = (\omega_i^*)^{-\frac{1}{1-\mu_i}} \left(\frac{1-\alpha_i}{\alpha_i} \right)^{\frac{1}{1-\mu_i}} \bar{K} \tag{52}$$

We simplify (51) and (52) with the normalization $\tilde{\kappa} = 1$ and rearrange to get:

$$\frac{L_1^{a*} + L_2^{a*}}{M_1^* + M_2^*} = \frac{2\frac{\bar{L}}{\bar{K}} - \left(\frac{\omega_1^*}{\tilde{\omega}}\right)^{\frac{-1}{1-\mu_1}} - \left(\frac{\omega_2^*}{\tilde{\omega}}\right)^{\frac{-1}{1-\mu_2}}}{(1+\tilde{\omega})^{\frac{-1}{\mu_1}} \left\{ 1 + \omega_1^* \left(\frac{\omega_1^*}{\tilde{\omega}}\right)^{\frac{-1}{1-\mu_1}} \right\}^{1/\mu_1} + (1+\tilde{\omega})^{\frac{-1}{\mu_2}} \left\{ 1 + \omega_2^* \left(\frac{\omega_2^*}{\tilde{\omega}}\right)^{\frac{-1}{1-\mu_2}} \right\}^{1/\mu_2}} \tag{53}$$

We solve for the price ratio in each country by plugging the expressions for the LHS and the RHS into (47) and plugging the expression of the factor price ratio as a function of the factor price ratio in the other country. To simplify notation, we define $\Omega_i = (\omega_i^*)^{\frac{\mu_i}{1-\mu_i}} + \tilde{\omega}^{\frac{1}{1-\mu_i}}$.

For the high-μ country we get:

$$\frac{2\frac{\bar{L}}{\bar{K}} - \left(\frac{\omega_1^*}{\tilde{\omega}}\right)^{\frac{-1}{1-\mu_1}} - \tilde{\omega}^{\frac{1}{1-\mu_2}} \left[(1+\tilde{\omega})^{\frac{\mu_1-\mu_2}{\mu_1(1-\mu_2)}} \Omega_1^{\frac{\mu_2(1-\mu_1)}{\mu_1(1-\mu_2)}} - \tilde{\omega}^{\frac{1}{1-\mu_2}} \right]^{\frac{-1}{\mu_2}}}{(1+\tilde{\omega})^{\frac{-1}{\mu_1}} \left[1 + \omega_1^* \left(\frac{\omega_1^*}{\tilde{\omega}}\right)^{\frac{-1}{1-\mu_1}} \right]^{\frac{1}{\mu_1}} + (1+\tilde{\omega})^{\frac{-(1-\mu_1)}{\mu_1(1-\mu_2)}} \Omega_1^{\frac{(1-\mu_1)}{\mu_1(1-\mu_2)}} \left[(1+\tilde{\omega})^{\frac{\mu_1-\mu_2}{\mu_1(1-\mu_2)}} \Omega_1^{\frac{\mu_2(1-\mu_1)}{\mu_1(1-\mu_2)}} - \tilde{\omega}^{\frac{1}{1-\mu_2}} \right]^{\frac{-1}{\mu_2}}}$$
$$= c(1+\tilde{\omega})^{\frac{1}{\mu_1}} \Omega_1^{\frac{-(1-\mu_1)}{\mu_1}}$$

For the low-μ country we get:

$$\frac{2\frac{\bar{L}}{\bar{K}} - \left(\frac{\omega_2^*}{\tilde{\omega}}\right)^{\frac{-1}{1-\mu_2}} - \tilde{\omega}^{\frac{1}{1-\mu_1}} \left[(1+\tilde{\omega})^{\frac{\mu_2-\mu_1}{\mu_2(1-\mu_1)}} \Omega_2^{\frac{\mu_1(1-\mu_2)}{\mu_2(1-\mu_1)}} - \tilde{\omega}^{\frac{1}{1-\mu_1}} \right]^{\frac{-1}{\mu_1}}}{(1+\tilde{\omega})^{\frac{-1}{\mu_2}} \left[1 + \omega_2^* \left(\frac{\omega_2^*}{\tilde{\omega}}\right)^{\frac{-1}{1-\mu_2}} \right]^{\frac{1}{\mu_2}} + (1+\tilde{\omega})^{\frac{-(1-\mu_2)}{\mu_2(1-\mu_1)}} \Omega_2^{\frac{(1-\mu_2)}{\mu_2(1-\mu_1)}} \left[(1+\tilde{\omega})^{\frac{\mu_2-\mu_1}{\mu_2(1-\mu_1)}} \Omega_2^{\frac{\mu_1(1-\mu_2)}{\mu_2(1-\mu_1)}} - \tilde{\omega}^{\frac{1}{1-\mu_1}} \right]^{\frac{-1}{\mu_1}}}$$
$$= c(1+\tilde{\omega})^{\frac{1}{\mu_2}} \Omega_2^{\frac{-(1-\mu_2)}{\mu_2}}$$

Rearranging and simplifying this expression, the implicit solution for the high-μ country is:

$$F_1(\omega_1;\mu_1,\mu_2,\frac{\bar{L}}{\bar{K}},c) = c\,(\omega_1^*)^{-1} + (c+1)\left(\frac{\omega_1^*}{\tilde{\omega}}\right)^{\frac{-1}{1-\mu_1}} - 2\frac{\bar{L}}{\bar{K}} + \frac{\left[c(1+\tilde{\omega})^{\frac{\mu_1-\mu_2}{\mu_1(1-\mu_2)}}\Omega_1^{\frac{\mu_2(1-\mu_1)}{\mu_1(1-\mu_2)}} + \tilde{\omega}^{\frac{1}{1-\mu_2}}\right]}{\left[(1+\tilde{\omega})^{\frac{\mu_1-\mu_2}{\mu_1(1-\mu_2)}}\Omega_1^{\frac{\mu_2(1-\mu_1)}{\mu_1(1-\mu_2)}} - \tilde{\omega}^{\frac{1}{1-\mu_2}}\right]^{\frac{1}{\mu_2}}} = 0$$

Rearranging and simplifying this expression, the implicit solution for the low-μ country is:

$$F_2(\omega_2;\mu_1,\mu_2,\frac{\bar{L}}{\bar{K}},c) = c\,(\omega_2^*)^{-1} + (c+1)\left(\frac{\omega_2^*}{\tilde{\omega}}\right)^{\frac{-1}{1-\mu_2}} - 2\frac{\bar{L}}{\bar{K}} + \frac{\left[c(1+\tilde{\omega})^{\frac{\mu_2-\mu_1}{\mu_2(1-\mu_1)}}\Omega_2^{\frac{\mu_1(1-\mu_2)}{\mu_2(1-\mu_1)}} + \tilde{\omega}^{\frac{1}{1-\mu_1}}\right]}{\left[(1+\tilde{\omega})^{\frac{\mu_2-\mu_1}{\mu_2(1-\mu_1)}}\Omega_2^{\frac{\mu_1(1-\mu_2)}{\mu_2(1-\mu_1)}} - \tilde{\omega}^{\frac{1}{1-\mu_1}}\right]^{\frac{1}{\mu_1}}} = 0$$

The first two terms replicate the analogous expression for the autarky equilibrium (30) while the third term now takes into account factor endowments in both countries. The fourth term is specific to the FTE: it accounts for the difference in capital-labor substitutability.

We can rewrite these expressions as a function of σ. In the high-σ country we get:

$$F_1(\cdot) = c\,(\omega_1^*)^{-1} + (c+1)\left(\frac{\omega_1^*}{\tilde{\omega}}\right)^{-\sigma_1} - 2\frac{\bar{L}}{\bar{K}} + \frac{\left[c(1+\tilde{\omega})^{\frac{\sigma_1-\sigma_2}{\sigma_1-1}}\left[(\omega_1^*)^{\sigma_1-1}+\tilde{\omega}^{\sigma_1}\right]^{\frac{\sigma_2-1}{\sigma_1-1}} + \tilde{\omega}^{\sigma_2}\right]}{\left[(1+\tilde{\omega})^{\frac{\sigma_1-\sigma_2}{\sigma_1-1}}\left[(\omega_1^*)^{\sigma_1-1}+\tilde{\omega}^{\sigma_1}\right]^{\frac{\sigma_2-1}{\sigma_1-1}} - \tilde{\omega}^{\sigma_2}\right]^{\frac{\sigma_2}{\sigma_2-1}}} = 0$$

In the low-σ country we get:

$$F_2(\cdot) = c\,(\omega_2^*)^{-1} + (c+1)\left(\frac{\omega_2^*}{\tilde{\omega}}\right)^{-\sigma_2} - 2\frac{\bar{L}}{\bar{K}} + \frac{\left[c(1+\tilde{\omega})^{\frac{\sigma_2-\sigma_1}{\sigma_2-1}}\left[(\omega_2^*)^{\sigma_2-1}+\tilde{\omega}^{\sigma_2}\right]^{\frac{\sigma_1-1}{\sigma_2-1}} + \tilde{\omega}^{\sigma_1}\right]}{\left[(1+\tilde{\omega})^{\frac{\sigma_2-\sigma_1}{\sigma_2-1}}\left[(\omega_2^*)^{\sigma_2-1}+\tilde{\omega}^{\sigma_2}\right]^{\frac{\sigma_1-1}{\sigma_2-1}} - \tilde{\omega}^{\sigma_1}\right]^{\frac{\sigma_1}{\sigma_1-1}}} = 0$$

4 Microfoundation: perceived capital-labor substitutability

In this section we propose a microfoundation of differences in measured capital-labor substitutability. We assume that the underlying production function in the routine sector is the same in both countries, and we model an innovation process that enables firms to increase factor efficiency in routine input production. Successful innovation is associated to a reorganization of the production process whereby a fraction of labor may be displaced from routine to non-routine tasks. This displacement is costly for workers. Our stylized model illustrates how differences in the institutional environment or in the perceived cost of labor reallocation may lead firms to select different innovation strategies.

Starting from any given reference point where the two countries have the same factor allocation to routine and non-routine tasks, we show that firms in the country with stronger organizational frictions choose to invest in less disruptive technology because failure in bargaining with workers over the implementation of new technology is more costly when the innovation step is bigger. An incremental innovation strategy means that more workers are retained in routine tasks, and the relative cost of labor in equilibrium is higher. At the aggregate level, the perceived sensitivity of the capital-labor ratio to changes in the relative cost of labor are smaller.

4.1 Benchmark: monopolistic competition in routine input production

4.2 Set-up with innovation policy but without layoff costs

4.3 Set-up with innovation policy and bargaining over labor reallocation

4.4 Building blocks of model

1. The R&D process (step 1: deriving the technology menu)[14]

   In order to produce, firms in the routine sector invest in R&D[15] and get N draws from the distribution of ideas about how to best use capital and labor in routine input production (factor efficiency). We follow Growiec and assume that these ideas are about elements of a complex production technology where the weakest link determines the overall productivity of the factor. So each firm in each period retains information on the weakest element across the set of its draws. The upside is that ideas are non-proprietory in this set-up so each firm gets information on the distribution of these weakest links across the full set of firms: this approach delivers a Weibull distribution of efficiency draws for each production factor across the full set of firms in routine production.

   To determine the menu of efficiency choices for labor and capital that is effectively available in each period we follow Growiec and posit that an efficiency choice is effectively available if and only if some fraction of firms has drawn this level of efficiency. The menu of available efficiency choices shifts rightward over time because sufficiently many firms attain efficiency that was previously drawn by just a few firms. This process takes place for improving the efficiency of each production factor in routine input production.

   The combination of available efficiency draws for both factors at any point in time determines the technology menu. For any given shape (α) and scale ($\lambda_{\{k,l\}}$) parameter of the Weibull distribution: given previous distribution of efficiency choices (k,l), we get new distribution of efficiency choices (k',l'). We pin down which (k,l) combinations are available by parameterizing technology menu as in Growiec: we choose the threshold above which we consider that an idea is 'available': available ideas are those that have been discovered by some threshold share of firms. For any given threshold probability kept constant across all periods ($T = -ln\Pr(\tilde{k} > k, \tilde{l} > l)$), new (k,l) combinations are those that have passed the threshold and are deemed 'available'.

   For Weibull distribution, this probability is: $\Pr(\tilde{k} > k, \tilde{l} > l) = \exp(-(l/\lambda_l)^\alpha - (k/\lambda_k)^\alpha)$ so we can solve for l as function of k and parameters: $l = \left[T\lambda_l^\alpha - (\frac{\lambda_l}{\lambda_k})^\alpha k^\alpha\right]^{1/\alpha}$ where α

[14] I stick to Growiec here but we could try to work out something a bit more intuitive and maybe simpler for the innovation process at a later stage?

[15] We think of this investment as a fixed cost paid in units of final output (its price corresponds to the consumer price index) because we don't want to change relative return to capital and to labor in each task.

is the shape parameter of the Weibull distribution, T is the chosen threshold for an idea to become available, and λ_l and λ_k are the location parameters of the Weibull distributions of labor (capital) efficiency. The location of the distribution shifts rightward as efficiency-augmenting R&D materializes. We get a menu (l,k) with a trade-off between available labor and capital efficiency.

The menu shifts upward as λ_l grows; the slope of the menu changes over time if the growth rate of expected efficiencies is factor-specific: $\lambda_l/\lambda_k \neq 1$. In the simple case where $\lambda_l/\lambda_k = 1$, any given choice of (α, T) translates into a set of upward-shifting isocurves in λ_l. A higher T means that we choose a smaller proba threshold for ideas to be deemed available, so all isocurves shift up with T for given α. For given T, a higher α means that trade-offs are less pronounced: isocurve flattens out: if distribution from which ideas are drawn is less dispersed, it is more likely that a small change in the efficiency of one factor is associated to a small change in the efficiency of the other factor. A higher α also means that upward shifts linked to growth in λ_l are less pronounced: the size of the innovative step is smaller when α is high, so the technology menu shifts by less when the location parameter increases.

The new element that we add to the process of innovation in Growiec is that the shape parameter α is itself a choice variable. Specifically, we follow Bonfiglioli, Crino, Gancia (2016) and link the size of the investment - which is a decision variable for the firm - to the degree of dispersion of efficiency draws: a higher investment translates into a reduction in the shape parameter of the ideas distribution.[16]

ADAPT EQUATIONS IN BCG TO GENERATE WEIBULL INSTEAD OF PARETO

I suggest that as a starting point we use the idea that the size of the investment translates into the expected size of the innovative step (expected change in the efficiency of each factor between period 1 and period 2) to model our mechanism of interest.[17] We can work with a simpler set-up where there are only two options: a better efficiency for some factor becomes available with some exogenous probability or it doesn't become available.

[16] This approach differs from canonical models of innovation such as Grossman and Helpman (1989) and Klette and Kortum (2004) in which investment in R&D increases the probability of successful innovation (the arrival rate of ideas) while the degree of dispersion of ideas (the size of the innovative step) is either modelled as an exogenous parameter common to all firms (GH) or is modelled as an exogenous distribution of firm-level ability to translate ideas into a flow of profits (KK).

[17] Suppose all firms can either innovate or not, and if they do the size of the innovation step is fixed. A bigger innovation step mechanically translates into higher variance in efficiency in the set of firms.

The size of the innovative step is increasing in the size of the R&D investment. We get a simple transition matrix where with some probability no factor is improved, with some probability one factor is improved, and with some probability both are improved. In each case we can solve for the optimal use of capital and labor in production (if there were no problems in adjusting factor quantities) and compute the number of workers that would need to transition from routine to non-routine occupations. We can also compute the increase in productivity in routine production associated to each situation.[18]

2. Technology adoption (step 2: determine best technology and associated surplus)

   To simplify the set of firm-level technology choices in the full set-up[19], we think that a firm in routine production makes a choice between the technology it currently uses and the implementation of the best alternative available technology.[20] We start from the point of normalization where countries do not differ in relative factor prices (we have to model how (k,l) is chosen in both countries at the point of normalization). We take technology menu as given at the point of normalization, and we investigate how firms react to change in technology menu (capital endowment also changes... we will have to incorporate that process as well..). So given α and T, we solve for the change in the technology menu using step 1,[21] and we look for (k',l') in the updated technology menu that would allow firm to improve its performance. For any given menu, we determine choice of best technology (k',l') by solving firm cost minimization problem while considering that the firm can freely choose the amount of capital and the number of workers (no negotiation hurdles). We continue to impose the constraint that both capital and labor markets clear, and both factors are fully employed.

   PROBLEM OF THE FIRM SIMILAR TO INITIAL MODELS OF ENDOG TECH: MONOP COMP FIRMS PRODUCE INPUTS; INVEST IN INNOVATION; EXPECTED PROFITS EQUAL TO COST OF ENTRY (FREE ENTRY)

[18] This approach is expected to convey the intuition for why negotiation may be more costly when the size of innovative step is bigger. We can always generalize to the full set of choices and compute the associated changes in workers allocated to each task.

[19] in the simplified set-up there are 4 outcomes to consider: no upgrade; one factor upgrade; both factors upgrade

[20] This alternative technology is not necessarily 'new' b/c the firm may have refrained from upgrading technology in the previous period; we only posit that the alternative technology is always one of the (k,l) combinations on the technology frontier.

[21] We know menu H(k',l') for any choice of α because the shift in the scale parameter is predetermined (growth rate of λ_l can either be function of parameter α or be normalized and independent of α: see BCG); and T is taken as given.

3. Negotiation and decision on how much to invest in R&D (step 3: pin down α)

   Having solved for the choice of the best alternative technology (given α), the firm knows how many workers become redundant (relatively to the previous technology choice). Further, the firm knows which surplus it is able to generate through the technology upgrade (see Acemoglu and Restrepo, NBER 2016 on this point). To successfully implement this new technology, the firm must negotiate with workers who need to be laid off *and reallocated to the non-routine sector*. In this model there is no unemployment, and workers are paid the same wage everywhere. So what makes workers reluctant to move must be the transition cost which we think of as country-specific. This cost can be modelled as a reduction in utility from re-training or as a firing cost imposed by the government or as higher bargaining power of workers associated to regulatory hurdles specific to lay-offs or any other country-specific characteristic. Countries may differ in the magnitude of this transition cost per worker or in the degree of convexity of this transition cost in the fraction of workers that need to be fired?

   Objective is to show that the risk of failure in the negotiation is increasing in the fraction of workers that need to be fired, that the fraction of workers who need to be fired is increasing in the size of the innovative step *decreasing in α*, and that firms in the country with higher (more convex) transition costs optimally choose a higher α (smaller size of investment) to reduce the risk of failure in negotiation.

4. Solving the model in autarky (step 4: get factor prices)

   Once we pin down α, we determine the change in the technology menu, the best alternative technology choice, routine output, labor allocation to tasks, and prices of factors and of final goods whereby we pin down the value of the investment (f) and the amount of final goods that are left for consumption once research effort has been paid for. We can then check whether the relative price of output in the non-routine-intensive sector is effectively lower in the low-α *high-μ* country.

5. Solving the model in free trade (step 5: check that trade does not overturn ranking of α)

   Free trade means that prices are now globally determined, so firms in routine production in each country may reconsider size of optimal investment in R&D *choice of α*.

4.5 Partial equilibrium in autarky

Using results in Growiec (2013, 2015) we start from a factor-augmenting process of technology upgrading. Firms choose labor efficiency and capital effiency from a technology menu which is updated each period because firms invest in new technology, and this investment materializes through new ideas drawn from some distribution. As shown by **?**, if any technology is made up of many components that are perfect complements, the distribution of ideas for new technologies is Weibull. From Bonfiglioli et al. (2015) we borrow the idea that the size of the investment determines whether firms source technology from a highly dispersed or little dispersed distribution of ideas, but instead of a Pareto, we assume a Weibull distribution of ideas.

4.6 General equilibrium in autarky

4.7 Opening up to trade

5 Estimation

6 Conclusion

References

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Appendices

A Characterization of the average cost function

A.1 AC minimization is the only solution to the macro problem

We can show that the choice of routine input production at which the average cost function is minimized provides the unique solution to the inner and outer problem defined in sec.3.1. To see this, consider any point on the marginal cost function (7) in choosing the quantity of the routine input M_i. The FOC for the inner problem is $MC_i^m = P_i^m$. Further, we know from the outer problem that (27) must hold. Hence, we combine (7) and (27), simplify the resulting expression for w_i and M_i, and use $c = \sum_n \theta_n(1-\beta_n)/\sum_n \theta_n \beta_n$ to get the first expression for total labor allocated to non-routine tasks:

$$L_i^a \;=\; c\,(1-\alpha_i)^{-\frac{1}{\mu_i}} \left(\frac{M_i}{A_i}\right)^{\mu_i} \left\{ \left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i \bar{K}^{\mu_i} \right\}^{\frac{1-\mu_i}{\mu_i}} \tag{54}$$

Labor market clearing together with the production technology for the routine input (5) provides the second expression for total labor allocated to non-routine tasks:

$$L_i^a = \bar{L} - L_i^m = \bar{L} - (1-\alpha_i)^{-\frac{1}{\mu_i}} \left\{ \left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i \bar{K}^{\mu_i} \right\}^{\frac{1}{\mu_i}} \tag{55}$$

Combining these two expressions gives:

$$\bar{L} - (1-\alpha_i)^{-\frac{1}{\mu_i}} \left\{ \left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i \bar{K}^{\mu_i} \right\}^{\frac{1}{\mu_i}} = c\,(1-\alpha_i)^{-\frac{1}{\mu_i}} \left(\frac{M_i}{A_i}\right)^{\mu_i} \left\{ \left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i \bar{K}^{\mu_i} \right\}^{\frac{1-\mu_i}{\mu_i}} \tag{56}$$

Rearranging to factor out $(1-\alpha_i)^{-\frac{1}{\mu_i}} \left\{ \left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i \bar{K}^{\mu_i} \right\}^{\frac{1}{\mu_i}}$ gives:

$$\bar{L} \;=\; (1-\alpha_i)^{-\frac{1}{\mu_i}} \left\{ \left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i \bar{K}^{\mu_i} \right\}^{\frac{1}{\mu_i}} \left[1 + \frac{c\left(\frac{M_i}{A_i}\right)^{\mu_i}}{\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i \bar{K}^{\mu_i}} \right] \tag{57}$$

Rearranging the term in square brackets gives:

$$\bar{L} \;=\; (1-\alpha_i)^{-\frac{1}{\mu_i}} \left\{ \left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i \bar{K}^{\mu_i} \right\}^{\frac{1-\mu_i}{\mu_i}} \left[(1+c)\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i \bar{K}^{\mu_i} \right] \tag{58}$$

The above expression indicates that for any given choice of endowments $\{\bar{L}, \bar{K}\}$ and parameters $\{\mu_i, c, \alpha_i, A_i\}$, there is a unique choice of routine input production that verifies FOCs for the inner and outer problems simultaneously. We have shown in 3.1.3 that routine input production M_i^* that verifies average cost minimization is compatible with any feasible choice of parameters and endowments. It follows that M_i^* must verify (58). We can confirm this assertion by plugging (14) into (58):

$$\bar{L} = (1-\alpha_i)^{-\frac{1}{\mu_i}} \alpha_i \bar{K}^{\mu_i} \left[\left(\frac{w_i}{r_i}\right)^{-\frac{\mu_i}{1-\mu_i}} \left(\frac{1-\alpha_i}{\alpha_i}\right)^{\frac{1}{1-\mu_i}} \right]^{\frac{1-\mu_i}{\mu_i}} \left\{ (1+c) \left[\left(\frac{w_i}{r_i}\right)^{-\frac{\mu_i}{1-\mu_i}} \left(\frac{1-\alpha_i}{\alpha_i}\right)^{\frac{1}{1-\mu_i}} + 1 \right] - 1 \right\}$$

Simplifying and rearranging gives:

$$\frac{w_i \bar{L}}{r_i \bar{K}} = (1+c) \left[1 + \left(\frac{w_i}{r_i}\right) \left(\frac{w_i/(1-\alpha_i)}{r_i/\alpha_i}\right)^{-\frac{1}{1-\mu_i}} \right] - 1 \tag{59}$$

We plug the value of the effective labor cost (**??**) into (59) and find that the LHS and the RHS are indeed equal:

$$\frac{w_i \bar{L}}{r_i \bar{K}} = \frac{w_i \bar{L}}{r_i \bar{K}} - c + (1+c) - 1 \tag{60}$$

Consequently, M_i^* in (14) constitutes the unique solution to the inner and outer problems. We expect to obtain the same solution if we solve the firm problem instead of the macro problem. The idea behind the proof is the following: there is a unique solution of the relative factor price for any given set of parameters that satisfies inner and outer cost minimization. As the production function is CRS, it must be the case that the ratio $\bar{K}/L_i^{m*}$ remains unchanged if we split production among N firms rather than concentrate it within a unique firm $N(\bar{K}/N)/(L_i^{m*}/N)$. Further, it must be the case that for any given factor price ratio w_i/r_i, routine labor and capital are combined in the same way in routine input production in both sectors. Consequently, the solution of the firm problem and of the macro problem must coincide.

A.2 MC and AC curvature

The impact of differences in capital-routine labor substitutability on resource allocation can be better understood by examining the curvature and the relative position of the MC and AC functions.

The marginal cost of production is 0 when $M_i \leq A_i \alpha_i^{\frac{1}{\mu_i}} \bar{K} = M_{min}$ since no labor is used in production up to that point. The marginal cost function is everywhere increasing for $M_i \geq M_{min}$:

$$\frac{dMC_i^m(\cdot)}{dM_i} = w_i (1-\mu_i)(1-\alpha_i)^{-\frac{1}{\mu_i}} \alpha_i \left(\frac{\bar{K}}{A_i}\right)^{\mu_i} M_i^{\mu_i - 2} \left[\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i \bar{K}^{\mu_i} \right]^{\frac{(1-2\mu_i)}{\mu_i}} > 0$$

We denote $j = w_i(1-\mu_i)(1-\alpha_i)^{-\frac{1}{\mu_i}}\alpha_i\left(\frac{\bar{K}}{A_i}\right)^{\mu_i}$. The second derivative of the marginal cost function is:

$$\frac{d^2MC_i^m(\cdot)}{dM_i{}^2} = jM_i^{\mu_i-3}\left[\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i\bar{K}^{\mu_i}\right]^{\frac{1-3\mu_i}{\mu_i}}\left\{(2-\mu_i)\alpha_i\bar{K}^{\mu_i} - (1+\mu_i)\left(\frac{M_i}{A_i}\right)^{\mu_i}\right\}$$

The sign of the second derivative is determined by the sign of the expression in the curly brackets. The marginal cost function is convex as long as:

$$(2-\mu_i)\alpha_i\bar{K}^{\mu_i} \geq (1+\mu_i)\left(\frac{M_i}{A_i}\right)^{\mu_i} \Leftrightarrow M_i \leq \left(\frac{2-\mu_i}{1+\mu_i}\right)^{\frac{1}{\mu_i}} A_i\alpha_i^{\frac{1}{\mu_i}}\bar{K} \tag{61}$$

The marginal cost function is convex at the intersection with the AC curve whenever:

$$M_i^* = A_i\alpha_i^{\frac{1}{\mu_i}}\bar{K}\left[1+\left(\frac{w_i}{r_i}\right)^{-\frac{\mu_i}{1-\mu_i}}\left(\frac{1-\alpha_i}{\alpha_i}\right)^{\frac{1}{1-\mu_i}}\right]^{1/\mu_i} \leq \left(\frac{2-\mu_i}{1+\mu_i}\right)^{\frac{1}{\mu_i}} A_i\alpha_i^{\frac{1}{\mu_i}}\bar{K}$$

$$\left[1+\left(\frac{w_i}{r_i}\right)^{-\frac{\mu_i}{1-\mu_i}}\left(\frac{1-\alpha_i}{\alpha_i}\right)^{\frac{1}{1-\mu_i}}\right] \leq \frac{2-\mu_i}{1+\mu_i} \tag{62}$$

There are three possible cases. If $\mu_i \geq .5$, the expression on the RHS is smaller than one, whereby the marginal cost curve always becomes concave before or at the intersection with the average cost curve. Second, for any μ, the marginal cost curve becomes concave before the intersection with the average cost curve if the relative wage is sufficiently high $(w_i/r_i) > \varpi_i^{1/(1-\mu_i)}$. If none of the above holds, then it could be the case that the marginal cost function is still convex at the intersection with the AC curve.

We also characterize the curvature of the AC function:

$$\frac{d^2AC(\cdot)}{d(M_i)^2} = \frac{d}{d(M_i)}\left\{\frac{w_i(1-\alpha_i)^{-\frac{1}{\mu_i}}\alpha_i\bar{K}^{\mu_i}\left[\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i\bar{K}^{\mu_i}\right]^{\frac{1-\mu_i}{\mu_i}} - r_i\bar{K}}{M_i^2}\right\}$$
$$\frac{w_i(1-\alpha_i)^{-\frac{1}{\mu_i}}\alpha_i\bar{K}^{\mu_i}\left[\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i\bar{K}^{\mu_i}\right]^{\frac{1-2\mu_i}{\mu_i}}\left[2\alpha_i\bar{K}^{\mu_i} - (1+\mu_i)\left(\frac{M_i}{A_i}\right)^{\mu_i}\right] + 2r_i\bar{K}}{M_i^3} \tag{63}$$

A sufficient condition for the convexity of the AC function is:

$$2\alpha_i\bar{K}^{\mu_i} \geq (1+\mu_i)\left(\frac{M_i}{A_i}\right)^{\mu_i} \Leftrightarrow M_i \leq \left(\frac{2}{1+\mu_i}\right)^{\frac{1}{\mu_i}} A_i\alpha_i^{\frac{1}{\mu_i}}\bar{K}$$

It is immediate that the inflexion point of the AC function always lies to the right of the inflexion point for the MC function. Further, it can be shown that the numerator of (63) is always positive at M_i^* whereby it follows that the inflexion point of the AC curve is situated in the increasing portion of the AC function.

A.3 Relative position of AC and MC curves

The marginal cost function is equal to the average cost function when $M_i = M_i^*$. Given $MC_i^m(M_{min}) < AC_i^m(M_{min})$ together with $dMC_i^m(\cdot)/dM_i > 0$ and $dAC_i^m(\cdot)/dM_i < 0$ for $M_i \in [M_{min}, M_i^*]$, the marginal cost function is below the average cost function for $M_i \in [M_{min}, M_i^*]$.

It remains to be shown that the marginal cost function is everywhere above the average cost function for $M_i > M_i^*$. Both functions are increasing in this range.

$$MC(M_i) > AC(M_i) \;\Leftrightarrow$$

$$\frac{w_i}{(1-\alpha_i)^{\frac{1}{\mu_i}}}\left(\frac{M_i}{A_i}\right)^{\mu_i}\left[\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i \bar{K}^{\mu_i}\right]^{\frac{1-\mu_i}{\mu_i}} > \frac{w_i}{(1-\alpha_i)^{\frac{1}{\mu_i}}}\left[\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i \bar{K}^{\mu_i}\right]^{\frac{1}{\mu_i}} + r_i \bar{K}$$

We rearrange this expression, factor out common terms and simplify to get:

$$w_i (1-\alpha_i)^{-\frac{1}{\mu_i}} \alpha_i \bar{K}^{\mu_i}\left[\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i \bar{K}^{\mu_i}\right]^{\frac{1-\mu_i}{\mu_i}} > r_i \bar{K}$$

$$\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i \bar{K}^{\mu_i} > \left(\frac{w_i}{r_i}\right)^{-\frac{\mu_i}{1-\mu_i}} \alpha_i^{-\frac{\mu_i}{1-\mu_i}} (1-\alpha_i)^{\frac{1}{1-\mu_i}} \bar{K}^{\mu_i}$$

$$M_i > A_i \alpha_i^{\frac{1}{\mu_i}} \bar{K}\left[1 + \left(\frac{w_i}{r_i}\right)^{-\frac{\mu_i}{1-\mu_i}}\left(\frac{1-\alpha_i}{\alpha_i}\right)^{\frac{1}{1-\mu_i}}\right]^{\frac{1}{\mu_i}} = M_i^*$$

The latter expression indeed holds. We conclude that the marginal cost function is everywhere above the average cost function beyond the point of average cost minimization.

Further, we can show that the marginal cost function constitutes an asymptote of the average cost function when $M_i \to \infty$. Consider the ratio:

$$\frac{MC(M_i)}{AC(M_i)} = \frac{w_i (1-\alpha_i)^{-\frac{1}{\mu_i}}\left(\frac{M_i}{A_i}\right)^{\mu_i}\left[\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i \bar{K}^{\mu_i}\right]^{\frac{1-\mu_i}{\mu_i}}}{w_i (1-\alpha_i)^{-\frac{1}{\mu_i}}\left[\left(\frac{M_i}{A_i}\right)^{\mu_i} - \alpha_i \bar{K}^{\mu_i}\right]^{\frac{1}{\mu_i}} + r_i \bar{K}} \tag{64}$$

We evaluate this ratio at the point in which routine input production tends to its maximum:

$$\lim_{L_i^m \to \bar{L}} M_i(L_i^m) = A_i \left[(1-\alpha_i)\bar{L}^{\mu_i} + \alpha_i \bar{K}^{\mu_i}\right]^{1/\mu_i}$$

$$\frac{MC(M_i(\bar{L}))}{AC(M_i(\bar{L}))} = \frac{w_i (1-\alpha_i)^{-1} \bar{L}^{1-\mu_i}\left[(1-\alpha_i)\bar{L}^{\mu_i} - \alpha_i \bar{K}^{\mu_i}\right]}{w_i \bar{L} + r_i \bar{K}}$$

Consider the position of the two curves when $\bar{L} \to \infty$:

$$\lim_{\bar{L}\to\infty} \frac{MC(M_i(\bar{L}))}{AC(M_i(\bar{L}))} = \lim_{\bar{L}\to\infty} \frac{w_i (1-\alpha_i)^{-1} \bar{L}^{1-\mu_i}\left[(1-\alpha_i)\bar{L}^{\mu_i} - \alpha_i \bar{K}^{\mu_i}\right]}{w_i \bar{L} + r_i \bar{K}} \tag{65}$$

Using L'Hopital's rule, this limit can be evaluated by taking the derivative of the numerator and of the denominator wrt $\bar{L}$:

$$\lim_{\bar{L}\to\infty}\frac{MC\left(M_i(\bar{L})\right)}{AC\left(M_i(\bar{L})\right)}=\lim_{\bar{L}\to\infty}\left\{\mu_i+(1-\mu_i)\left[1+\frac{\alpha_i}{1-\alpha_i}\left(\frac{\bar{K}}{\bar{L}}\right)^{\mu_i}\right]\right\}=1 \tag{66}$$

The second term in the square brackets approaches 0 from above. Consequently, the AC function converges to the MC function from below as $M_i\to\infty$.

B Alternative approach to solving the benchmark model

B.1 The problem of the firm in routine input production

The cost minimization problem of the firm is:

$$\begin{cases}\text{Min } w_iL_i^m+r_iK_i\\ s.t.\ M_i\leq A_i\left[\alpha_i(K)_i^{\mu_i}+(1-\alpha_i)(L_i^m)^{\mu_i}\right]^{1/\mu_i}\end{cases}$$

The first order conditions define relative factor demand as a function of the factor price ratio:

$$\frac{L_i^m}{K_i} = \left[\frac{w_i}{r_i}\frac{\alpha_i}{1-\alpha_i}\right]^{-\frac{1}{1-\mu_i}} \tag{67}$$

We rearrange this expression to solve for each of the factors and plug it into the production function to obtain conditional factor demands:

$$K_i=\frac{M_i}{A_i}\left[\alpha_i+(1-\alpha_i)\left(\frac{w_i}{r_i}\frac{\alpha_i}{(1-\alpha_i)}\right)^{\frac{\mu_i}{\mu_i-1}}\right]^{-\frac{1}{\mu_i}}=\frac{M_i}{A_i}\left[\alpha_i\right]^{-\frac{1}{\mu_i}}\left[1+\left(\frac{w_i}{r_i}\right)^{\frac{\mu_i}{\mu_i-1}}\left(\frac{1-\alpha_i}{\alpha_i}\right)^{\frac{1}{1-\mu_i}}\right]^{-\frac{1}{\mu_i}}$$

$$L_i^m=\frac{M_i}{A_i}\left[(1-\alpha_i)+\alpha_i\left(\frac{w_i}{r_i}\frac{\alpha_i}{(1-\alpha_i)}\right)^{\frac{\mu_i}{1-\mu_i}}\right]^{-\frac{1}{\mu_i}}=\frac{M_i}{A_i}\left[1-\alpha_i\right]^{-\frac{1}{\mu_i}}\left[1+\left(\frac{w_i}{r_i}\right)^{\frac{\mu_i}{1-\mu_i}}\left(\frac{\alpha_i}{1-\alpha_i}\right)^{\frac{1}{1-\mu_i}}\right]^{-\frac{1}{\mu_i}}$$

We rearrange each of the expressions in square brackets. In the capital equation we factor out $\alpha_i^{-\frac{1}{1-\mu_i}}r_i^{\frac{\mu_i}{1-\mu_i}}$. In the routine labor equation we factor out $(1-\alpha_i)^{-\frac{1}{1-\mu_i}}w_i^{\frac{\mu_i}{1-\mu_i}}$.

For capital, we get:

$$\begin{aligned}K_i\left(M_i;w_i,r_i\right)&=\frac{M_i}{A_i}\left[\alpha_i\right]^{-\frac{1}{\mu_i}}\left[\alpha_i^{-\frac{1}{1-\mu_i}}r_i^{\frac{\mu_i}{1-\mu_i}}\right]^{-\frac{1}{\mu_i}}\left[\alpha_i^{\frac{1}{1-\mu_i}}r_i^{-\frac{\mu_i}{1-\mu_i}}+(1-\alpha_i)^{\frac{1}{1-\mu_i}}w_i^{\frac{-\mu_i}{1-\mu_i}}\right]^{-\frac{1}{\mu_i}}\\&=\frac{M_i}{A_i}\left[\frac{\alpha_i}{r_i}\right]^{\frac{1}{1-\mu_i}}\left[\alpha_i^{\frac{1}{1-\mu_i}}r_i^{-\frac{\mu_i}{1-\mu_i}}+(1-\alpha_i)^{\frac{1}{1-\mu_i}}w_i^{\frac{-\mu_i}{1-\mu_i}}\right]^{-\frac{1}{\mu_i}}\end{aligned}$$

For routine labor, we get:

$$\begin{aligned}L_i^m\left(M_i;w_i,r_i\right)&=\frac{M_i}{A_i}\left[1-\alpha_i\right]^{-\frac{1}{\mu_i}}\left[(1-\alpha_i)^{-\frac{1}{1-\mu_i}}w_i^{\frac{\mu_i}{1-\mu_i}}\right]^{-\frac{1}{\mu_i}}\left[\alpha_i^{\frac{1}{1-\mu_i}}r_i^{-\frac{\mu_i}{1-\mu_i}}+(1-\alpha_i)^{\frac{1}{1-\mu_i}}w_i^{\frac{-\mu_i}{1-\mu_i}}\right]^{-\frac{1}{\mu_i}}\\&=\frac{M_i}{A_i}\left[\frac{1-\alpha_i}{w_i}\right]^{\frac{1}{1-\mu_i}}\left[\alpha_i^{\frac{1}{1-\mu_i}}r_i^{-\frac{\mu_i}{1-\mu_i}}+(1-\alpha_i)^{\frac{1}{1-\mu_i}}w_i^{\frac{-\mu_i}{1-\mu_i}}\right]^{-\frac{1}{\mu_i}}\end{aligned} \tag{68}$$

We plug the conditional factor demands in the cost of production to obtain the unit cost function:

$$C(w_i,r_i;1)=\frac{1}{A_i}\left[\alpha_i^{\frac{1}{1-\mu_i}}r_i^{-\frac{\mu_i}{1-\mu_i}}+(1-\alpha_i)^{\frac{1}{1-\mu_i}}w_i^{-\frac{\mu_i}{1-\mu_i}}\right]^{-\frac{1}{\mu_i}}\left[(1-\alpha_i)^{\frac{1}{1-\mu_i}}w_i^{1-\frac{1}{1-\mu_i}}+\alpha_i^{\frac{1}{1-\mu_i}}r_i^{1-\frac{1}{1-\mu_i}}\right]$$

$$C(w_i,r_i;1)=\frac{1}{A_i}\left[\alpha_i^{\frac{1}{1-\mu_i}}r_i^{-\frac{\mu_i}{1-\mu_i}}+(1-\alpha_i)^{\frac{1}{1-\mu_i}}w_i^{-\frac{\mu_i}{1-\mu_i}}\right]^{\frac{\mu_i-1}{\mu_i}} \tag{69}$$

The price of the routine input coincides with the solution of the macro problem (18).

B.2 Autarky Equilibrium

The resource constraint on capital puts an upper bound on the maximal amount of the routine input that can be produced by optimally combining capital and labor:

$$K_i^* = \frac{M_i}{A_i}\left[\frac{\alpha_i A_i P_i^m}{r_i}\right]^{\frac{1}{1-\mu_i}} \le \bar{K} \Leftrightarrow M_i^* \le \bar{K}A_i\left[\frac{\alpha_i A_i P_i^m}{r_i}\right]^{-\frac{1}{1-\mu_i}} \tag{70}$$

The level of production of the routine input coincides with the solution of the macro problem whenever the capital endowment is fully used ((71) replicates (14)):

$$M_i^* = \bar{K}A_i\left[\frac{\alpha_i}{r_i}\right]^{-\frac{1}{1-\mu_i}}\left[\alpha_i^{\frac{1}{1-\mu_i}}r_i^{-\frac{\mu_i}{1-\mu_i}}+(1-\alpha_i)^{\frac{1}{1-\mu_i}}w_i^{-\frac{\mu_i}{1-\mu_i}}\right]^{\frac{1}{\mu_i}}$$

$$M_i^* = \bar{K}A_i\alpha_i^{\frac{1}{\mu_i}}\left[\alpha_i^{-\frac{1}{\mu_i(1-\mu_i)}}r_i^{\frac{1}{1-\mu_i}}\right]\left[\alpha_i^{\frac{1}{1-\mu_i}}r_i^{-\frac{\mu_i}{1-\mu_i}}+(1-\alpha_i)^{\frac{1}{1-\mu_i}}w_i^{-\frac{\mu_i}{1-\mu_i}}\right]^{\frac{1}{\mu_i}}$$

$$M_i^* = \bar{K}A_i\alpha_i^{\frac{1}{\mu_i}}\left[1+\left(\frac{w_i}{r_i}\right)^{-\frac{\mu_i}{1-\mu_i}}\left(\frac{1-\alpha_i}{\alpha_i}\right)^{\frac{1}{1-\mu_i}}\right]^{\frac{1}{\mu_i}} \tag{71}$$

The above expression uses capital market clearing to define the optimal quantity of the routine output. We obtain the second expression for the optimal quantity of the routine output using labor market clearing. Cost minimization in the production of final goods delivers (27). We use labor market clearing to write $L_i^{a*}=\bar{L}-L_i^{m*}(M_i^*)$. We use conditional labor demand in routine output production (68) to replace L_i^{m*} by its value whereby (27) becomes:

$$\frac{\bar{L}-\frac{M_i^*}{A_i}\left[\frac{1-\alpha_i}{w_i}\right]^{\frac{1}{1-\mu_i}}\left[\alpha_i^{\frac{1}{1-\mu_i}}r_i^{-\frac{\mu_i}{1-\mu_i}}+(1-\alpha_i)^{\frac{1}{1-\mu_i}}w_i^{\frac{-\mu_i}{1-\mu_i}}\right]^{-\frac{1}{\mu_i}}}{M_i^*} = c\frac{P_i^m}{w_i} \tag{72}$$

We factor out M_i^* on the LHS, replace P_i^m by its value in (69) on the RHS, rearrange the expression and solve for $M_i^*$to get:

$$M_i^* = A_i w_i \bar{L}\frac{\left[\alpha_i^{\frac{1}{1-\mu_i}}r_i^{-\frac{\mu_i}{1-\mu_i}}+(1-\alpha_i)^{\frac{1}{1-\mu_i}}w_i^{\frac{-\mu_i}{1-\mu_i}}\right]^{\frac{1}{\mu_i}}}{\left[c\alpha_i^{\frac{1}{1-\mu_i}}r_i^{-\frac{\mu_i}{1-\mu_i}}+(1+c)(1-\alpha_i)^{\frac{1}{1-\mu_i}}w_i^{\frac{-\mu_i}{1-\mu_i}}\right]} \tag{73}$$

We equate (73) with the first line of (71) to get:

$$\bar{K}\left[\frac{\alpha_i}{r_i}\right]^{-\frac{1}{1-\mu_i}} = w_i\bar{L}\left[c\alpha_i^{\frac{1}{1-\mu_i}}r_i^{-\frac{\mu_i}{1-\mu_i}}+(1+c)(1-\alpha_i)^{\frac{1}{1-\mu_i}}w_i^{\frac{-\mu_i}{1-\mu_i}}\right]^{-1}$$

We factor out $\alpha_i^{1/(1-\mu_i)}r_i^{-\mu_i/(1-\mu_i)}$ and simplify the above expression to obtain an implicit solution for the factor price ratio $\omega_i^* = (w_i/r_i)^*$:

$$(\omega_i^*)^{-1}c+(1+c)\left[\frac{1-\alpha_i}{\alpha_i}\right]^{\frac{1}{1-\mu_i}}(\omega_i^*)^{-\frac{1}{1-\mu_i}}-\frac{\bar{L}}{\bar{K}}=0 \tag{74}$$

The solution to the autarky equilibrium is unchanged: (74) replicates (30).

B.3 Existence and uniqueness of the solution

As we show in 3.1.3, the polynomial in (74) has a unique positive root ω_i^* whenever the relative price of capital is not 'too high' $(r_i/w_i)^* \leq c^{-1}(\bar{L}/\bar{K})$. To investigate whether this inequality always holds, we start from some initial endowments for which it is satisfied and characterize the magnitude of the change in the factor price ratio and in the relative endowment following a positive shock to $\bar{L}/\bar{K}$.[22] Differentiating both sides with respect to $\bar{L}/\bar{K}$, we get:

$$\left[\frac{\partial\left(\frac{r_i}{w_i}\right)^*}{\partial\left(\frac{\bar{L}}{\bar{K}}\right)}\right]d\left(\frac{\bar{L}}{\bar{K}}\right)=\frac{1}{1+\sigma_i\left[\frac{w_i^*\bar{L}}{r_i^*\bar{K}}-1\right]}d\left(\frac{\bar{L}}{\bar{K}}\right)\leq\frac{1}{c}d\left(\frac{\bar{L}}{\bar{K}}\right)\;\Leftrightarrow\; c\leq 1+\sigma_i\left[\frac{w_i^*\bar{L}}{r_i^*\bar{K}}-1\right] \tag{75}$$

As long as the above inequality holds, the change in the factor price ratio is smaller than the change in relative factor endowments, and the initial inequality continues to hold. The magnitude of c depends on factor shares in production of final goods and on the shares of these final goods in consumption. For simplicity, we assume that $c = 1$.[23] It is immediate that the initial inequality can be rearranged as $1 \leq w_i^*\bar{L}/r_i^*\bar{K}$ whereby (75) is verified. It follows that the polynomial has a unique positive solution for any $\bar{L}'/\bar{K}' \geq \bar{L}/\bar{K}$, i.e. both labor and capital continue to be used in routine input production as labor becomes more and more abundant.

The intuition is the following. An increase in the labor endowment translates into an increase in the relative cost of capital (75). Notwithstanding this increase in the cost of capital, it remains optimal to use the full amount of capital in routine input production. Indeed, (67) indicates that by increasing the amount of capital used in production we always decrease the relative cost of capital and free up labor for non-routine tasks. By freeing up labor from routine tasks,

[22] One such initial endowment point is simply $\bar{L}/\bar{K} = 1$.

[23] $c = 1$ if the two goods carry equal weight in consumption ($\theta_1 = \theta_2 = .5$) and $\beta_1 + \beta_2 = 1$.

we always increase the total quantity of final goods that can be produced, thereby making the consumer better off.

Next, we consider the change in relative endowments and in the relative factor price ratio following a positive shock to $(\bar{K}/\bar{L})$. For the initial endowments, the inequality $(w_i/r_i)^* \geq c(\bar{K}/\bar{L})$ is verified. Differentiating both sides with respect to $\bar{K}/\bar{L}$, we get:

$$\left[\frac{\partial\left(\frac{w_i}{r_i}\right)^*}{\partial\left(\frac{\bar{K}}{\bar{L}}\right)}\right] = \frac{\left(\frac{w_i^*\bar{L}}{r_i^*\bar{K}}\right)^2}{c + \frac{1}{1-\mu_i}\left\{(1+c)\left(\frac{1-\alpha_i}{\alpha_i}\right)^{\frac{1}{1-\mu_i}}\left[\left(\frac{w_i}{r_i}\right)^*\right]^{-\frac{\mu_i}{1-\mu_i}}\right\}} \geq c \tag{76}$$

From the polynomial we know that the expression in the curly brackets of (76) is equal to $[w_i\bar{L}/r_i\bar{K} - c]$. Rearranging and simplifying the above expression, we get:

$$\left[\frac{\partial\left(\frac{w_i}{r_i}\right)^*}{\partial\left(\frac{\bar{K}}{\bar{L}}\right)}\right] = \frac{(1-\mu_i)\left(\frac{w_i^*\bar{L}}{r_i^*\bar{K}}\right)^2}{\frac{w_i^*\bar{L}}{r_i^*\bar{K}} - \mu_i c} \geq c \tag{77}$$

Again we set $c = 1$, and simplify the above expression to get:

$$\frac{(1-\mu_i)\left(\frac{w_i^*\bar{L}}{r_i^*\bar{K}}\right)^2}{\frac{w_i^*\bar{L}}{r_i^*\bar{K}} - \mu_i} \geq 1 \quad \Leftrightarrow \quad \frac{w_i^*\bar{L}}{r_i^*\bar{K}} \geq \frac{\mu_i}{1-\mu_i} \tag{78}$$

As long as the above inequality holds, the change in the factor price ratio exceeds the change in relative factor endowments, and the initial inequality always holds. The above inequality is necessarily verified if $\mu_i \leq .5$. However, the inequality may be violated for $\mu_i > .5$ whereby the initial inequality may be violated for high enough μ and sufficiently abundant capital. The intuition is straightforward. As capital endowment increases, the use of labor in routine tasks becomes more and more expensive. If μ is sufficiently high, we may reach a situation where capital becomes sufficiently cheap to fully replace labor in routine tasks.

If one or both countries stop using labor in routine input production, its price becomes $P_i^m = r_i\bar{K}/M_i$ where $M_i = A_i\alpha_i^{1/\mu_i}\bar{K}$ whereby $P_i^m = r_i/A_i\alpha_i^{1/\mu_i}$. If this approach to production is cost-minimizing, it must be that the price of the routine input is lower without using labor:

$$\frac{r_i}{A_i\alpha_i^{1/\mu_i}} \leq \frac{r_i}{A_i\alpha_i^{1/\mu_i}}\left[1 + \frac{w_i}{r_i}\left(\frac{w_i/(1-\alpha_i)}{r_i/\alpha_i}\right)^{-\frac{1}{1-\mu_i}}\right]^{\frac{\mu_i-1}{\mu_i}} \Leftrightarrow$$

$$\left[1 + \left(\frac{w_i}{r_i}\right)^{-\frac{\mu_i}{1-\mu_i}}\left(\frac{1-\alpha_i}{\alpha_i}\right)^{\frac{1}{1-\mu_i}}\right]^{\frac{-(1-\mu_i)}{\mu_i}} \geq 1 \tag{79}$$

The LHS of (79) is strictly smaller than 1 as long as w_i/r_i is finite. The LHS converges to 1 when $w_i/r_i \to \infty$. We conclude that when capital endowment becomes sufficiently abundant and $\mu > .5$, the weight of labor in routine input production becomes negligible. In the latter case, $L_i^a \to \bar{L}'$, $M_i \to A_i \alpha_i^{1/\mu_i} \bar{K}'$, and $P_i^m = r_i / A_i \alpha_i^{1/\mu_i}$ whereby (27) becomes:

$$\frac{L_i^{a*}}{M_i^*} = c\frac{P_i^m}{w_i} \;\Leftrightarrow\; \frac{\bar{L}'}{\bar{K}'} = c\frac{r_i}{w_i} \;\Leftrightarrow\; \omega_i^* = c\frac{\bar{K}'}{\bar{L}'} \tag{80}$$

This situation must occur in the high-μ country before the low-μ country because the equilibrium factor price ratio $\omega_1^*(\mu_1) < \omega_2^*(\mu_2)$ when capital endowment increases relatively to the point of normalization. It follows that $\frac{\bar{K}'}{\bar{L}'}$ for which $\omega_1^*(\mu_1) \to c\frac{\bar{K}'}{\bar{L}'}$ has $\omega_2^*(\mu_2) > c\frac{\bar{K}'}{\bar{L}'}$. As the relative wage is lower in the high-μ country, this country continues to have a relatively lower autarky price for the non-routine intensive final good.

If $\mu_2 > .5$ and the capital endowment continues to increase, the low-μ country also reaches the point where only capital is used in routine input production. Beyond this threshold, differences in capital-labor substitutability cease to be a source of comparative advantage.

To sum up, we have a unique positive solution to the polynomial for any factor endowments if $\mu_i \leq .5$, and the pattern of specialization described in the core of the paper always holds. Whenever both μ_1 and μ_2 are strictly bigger than .5, there exists a threshold at which the relative capital endowment is sufficiently high for labor to become negligible in routine input production. In the latter case, our mechanism ceases to be a source of comparative advantage.